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AWARDS & APPRECIATION

Century International Quality Era Award (Geneva, Switzerland)
KRISHNA Group was honored with “Century International Quality Era Award”, for the year “2003”, in “Total Quality Management” category. This prestigious award was presented, at the “5th International Quality Convention”, by Mr. Jose E. Prito, President & CEO, Business Initiatives Directions (B.I.D.), Madrid (Spain). A few of the honorable citizens from more than 100 countries participated in this prestigious ceremony.

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KRISHNA Group was honored with the “Distinguished Publisher’s Award” for the year “2002”. This award was conferred to Mr. S.K. Rastogi, Managing Director, Krishna Group, for excellence in Book Production by F.E.P.I. (Federation of Educational Publishers in India). This award has been considered a benchmark in assessing National Book Production Standards.

Goel Publishing House “Estd. 1942”
It is one of the most reputed and oldest Publishing House for the books in Chemistry at Graduation and Post Graduation Level. The Publishing House has completed multiple decades of its Contribution to society and Success in delivering perfection.

KRISHNA Prakashan Media Pvt. Ltd. “Estd. 1960”
Initially Krishna Prakashan Mandir and now Krishna Prakashan Media (P) Ltd. is Global brand in Educational Publishing Sector. Krishna Prakashan is famous for publishing quality science, Technical & Management Text Books/ Reference books for graduate & post graduate students of all Indian universities.

gMASTERg Educorp “Estd. 2013”
gMASTERg Educorp, being the latest gem of “THE KRISHNA GROUP” started its operation with a vision to enlighten the world with its “Master Book” series of publications touching each and every aspect of life. As the latest entrant of KRISHNA Family, it is bound to rock the Publishing Sector with its Innovative approach in e-Publishing, e-Selling and eXports.
# Mathematics & Statistics

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- Inverse Circular Functions  
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- Some Important Results about Inverse Functions  
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- Angles in Arithmetical Progression.
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- Continuous functions and classification of discontinuities  
- Differentiability  
- Chain rule of Differentiability  
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- Taylor’s theorems with Lagrange’s and Cauchy’s forms of remainder  
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- Confocal conics  
- Polar equation of a conic and its properties  
- Three dimensional system of co-ordinates  
- Projection and direction cosines  
- Plane  
- Straight line  
- Sphere  
- Cone and cylinder  
- Central conicoids  
- Reduction of general equation of second degree  
- Tangent plane and normal to a conicoid  
- Pole and polar  
- Conjugate diameters  
- Generating lines  
- Plane sections.

**Vector Calculus**  
- A.R. Vasishtha & Others  

- Vector differentiation and integration  
- Gradient  
- Divergence and curl and their properties  
- Line integrals  
- Theorems of Gauss  
- Green and Stokes and problems based on these.

**Linear Algebra**  
- A.R. Vasishtha & Others  

- Vector spaces and their elementary properties  
- Subspaces  
- Linear dependence and independence  
- Basis and dimension  
- Direct sum  
- Quotient space  
- Linear transformations and their algebra  
- Range and null space  
- Rank and nullity  
- Matrix representation of linear transformations  
- Change of basis  
- Linear functionals  
- Dual space  
- Bi-dual space  
- Natural isomorphism  
- Annihilators  
- Bilinear and quadratic forms  
- Inner product spaces  
- Cauchy-Schwarz’s inequality  
- Bessel’s inequality and orthogonality.

**Matrices**  
- A.R. Vasishtha & Others  

- Symmetric and skew-symmetric matrices  
- Hermitian and skew-Hermitian matrices  
- Orthogonal and unitary matrices  
- Triangular and diagonal matrices  
- Rank of a matrix  
- Elementary transformations  
- Echelon and normal forms  
- Inverse of a matrix by elementary transformations  
- Characteristic equation  
- Eigen values and eigen vectors of a matrix  
- Cayley-Hamilton’s theorem and its use in finding inverse of a matrix  
- Application of matrices to solve a system of linear (both homogeneous and non-homogeneous) equations  
- Consistency and general solution  
- Diagonalization of square matrices with distinct eigen values  
- Quadratic forms.
Contents

Differential Equations

721-17 (B)  
• Formation of a differential equation (D.E.)  
• Degree, order and solution of D.E.  
• Equations of first order and first degree: Separation of variables method  
• Solution of homogeneous equations  
• Linear equations and exact equations  
• Linear differential equations with constant coefficients  
• Homogeneous linear differential equations  
• Differential equations of the first order but not of the first degree  
• Clairaut’s equations and singular solutions  
• Orthogonal trajectories  
• Simultaneous linear differential equations with constant coefficients  
• Linear differential equations of the second order (including the method of variation of parameters)  
• Series solutions of second order differential equations  
• Legendre and Bessel functions (\(P_n\) and \(J_n\) only) and their properties  
• Order, degree and formation of partial differential equations  
• Partial differential equations of the first order  
• Lagrange’s equations  
• Charpit’s general method  
• Linear partial differential equations with constant coefficients  
• Partial differential equations of the second order  
• Monge’s method.

Integral Transforms

722-17 (B)  
• The concept of transform  
• Integral transforms and kernel  
• Linearity property of transforms  
• Laplace transform  
• Inverse Laplace transform  
• Convolution theorem  
• Applications of Laplace transform to solve ordinary differential equations  
• Fourier transforms (finite and infinite)  
• Fourier integral  
• Applications of Fourier transform to boundary value problems  
• Fourier series.

Statics

723-13 (B)  
• Common catenary  
• Centre of gravity  
• Stable and unstable equilibrium  
• Virtual work  
• Forces in three dimensions  
• Poinsot’s central axis  
• Wrenches  
• Null line and null plane.

Dynamics

799-13  
• Velocity and acceleration along radial and transverse directions and along tangential and normal directions  
• Simple harmonic motion  
• Motion under other laws of forces  
• Earth attraction  
• Elastic strings  
• Motion in resisting medium  
• Constrained motion (circular and cycloidal only)  
• Motion on smooth and rough plane curves  
• Rocket motion  
• Central orbits and Kepler’s law  
• Motion of a particle in three dimensions.

Real Analysis

744-02 (B)  
• Axiomatic study of real numbers  
• Completeness property in \(\mathbb{R}\)  
• Archimedean property  
• Countable and uncountable sets  
• Neighbourhood  
• Interior points  
• Limit points  
• Open and closed sets  
• Derived sets  
• Dense sets  
• Perfect sets  
• Bolzano-Weierstrass theorem  
• Sequences of real numbers  
• Subsequences  
• Bounded and monotonic sequences  
• Convergent sequences  
• Cauchy’s theorems on limit  
• Cauchy sequence  
• Cauchy’s general principle of convergence  
• Uniform convergence of sequences and series of functions  
• Weierstrass M-test  
• Abel’s and Dirichlet’s tests  
• Sequential continuity  
• Boundness and intermediate value properties of continuous functions  
• Uniform continuity  
• Meaning of sign of derivative  
• Darboux theorem  
• Limit and continuity of functions of two variables  
• Taylor’s theorem for functions of two variables  
• Maxima and minima of functions of three variables  
• Lagrange’s method of undetermined multipliers  
• Riemann integral  
• Integrability of continuous and monotonic functions  
• Fundamental theorem of integral calculus  
• Mean value theorems of integral calculus  
• Improper integrals and their convergence  
• Comparison test  
• \(\mu\)-test  
• Abel’s test  
• Dirichlet’s test  
• Integral as a function of a parameter and its differentiability and integrability  
• Definition and examples of metric spaces  
• Neighbourhoods  
• Interior points  
• Limit points  
• Open and closed sets  
• Subspaces  
• Convergent and Cauchy sequences  
• Completeness  
• Cantor’s intersection theorem.
Complex Analysis

- Functions of a complex variable
- Concepts of limit
- Continuity and differentiability of complex functions
- Analytic functions
- Cauchy-Riemann equations (Cartesian and polar form)
- Harmonic functions
- Orthogonal system
- Power series as an analytic function
- Elementary functions
- Mapping by elementary functions
- Linear and bilinear transformations
- Fixed points
- Cross ratio
- Inverse points and critical points
- Conformal transformations
- Complex Integration
- Line integral
- Cauchy's fundamental theorem
- Cauchy's integral formula
- Morera's theorem
- Liouville theorem
- Maximum Modulus theorem
- Taylor and Laurent series
- Singularities and zeros of an analytic function
- Rouche's theorem
- Fundamental theorem of algebra
- Analytic continuation
- Residue theorem and its applications to the evaluation of definite integrals
- Argument principle.

Numerical Analysis & Programming in C

- Shift operator
- Forward and backward difference operators and their relationships
- Fundamental theorem of difference calculus
- Interpolation
- Newton-Gregory's forward and backward interpolation formulae
- Divided differences
- Newton's divided difference formula
- Lagrange's interpolation formula
- Central differences
- Formulæ based on central differences: Gauss, Striling's, Bessel's and Everett's interpolation formulæ
- Numerical differentiation
- Numerical integration
- General quadrature formula
- Trapezoidal and Simpson's rules
- Weddle's rule
- Cote's formula
- Numerical solution of first order differential equations: Euler's method
- Picard's method
- Runge-Kutta method and Milne's method
- Numerical solution of linear, homogeneous and simultaneous difference equations
- Generating function method
- Solution of transcendental and polynomial equations by iteration, bisection
- Regula-Falsi and Newton-Raphson methods
- Algebraic eigen value problems: Power method
- Jacobi's method
- Given's method
- Householder's method and Q-R method
- Approximation: Different types of approximations
- Least square polynomial approximation
- Polynomial approximation using orthogonal polynomials
- Legendre approximation
- Approximation with trigonometric functions
- Exponential functions
- Rational functions
- Chebyshev polynomials
- Programmer's model of computer
- Algorithms
- Data type
- Arithmetic and input/output instruction
- Decisions
- Control structures
- Decision statements
- Logical and conditional operators
- Loop case control structures
- Functions
- Recursion
- Preprocessors
- Arrays
- Puppetting of strings
- Structures
- Pointers
- File formatting.

Linear Programming

- Linear programming problems
- Statement and formation of general linear programming problems
- Graphical method
- Slack and surplus variables
- Standard and matrix forms of linear programming problem
- Basic feasible solution
- Convex sets
- Fundamental theorem of linear programming
- Simplex method
- Artificial variables
- Big-M method
- Two phase method
- Resolution of degeneracy
- Revised simplex method
- Sensitivity Analysis
- Duality in linear programming problems
- Dual simplex method
- Primal-dual method
- Integer programming
- Transportation problems
- Assignment problems.

Differential Geometry & Tensor Analysis

- Local theory of Curves: Space curves
- Examples
- Plane curves
- Tangent and normal and binormal
- Osculating plane
- Normal plane and rectifying plane
- Helices
- Serret-Frenet apparatus
- Contact between curve and surfaces
- Tangent surfaces
- Involutes and evolutes of curves
- Intrinsic equations
- Fundamental existence theorem for space curves
- Local theory of Surfaces-Parametric patches on surface curve of a surface
- Surfaces of revolutions
- Helicoids
- Metric-first fundamental form and arc length
- Local theory of surfaces (Contd.)
- Direction coefficients
- Families of curves
- Intrinsic properties
- Geodesics, canonical geodesic equations, normal properties of geodesics
- geodesics curvature, geodesics polars
- Gauss-Bonnet theorem
- Gaussian curvature
- Normal curvature
- Meusnier's theorem
- Mean curvature
- Gaussian curvature
- Umbilic points
- Lines of curvature
- Rodrigue's formula
- Euler's theorem
- The fundamental equation of surface theory
- The equation of Gauss, the equation of Weingarten, the Mainardi-Codazzi equation
- Tensor algebra: Vector spaces
- The dual spaces
- Tensor product of vector spaces
- Transformation formulæ
- Contraction
- Special tensor
- Inner product
- Associated tensor
- Differential Manifold-examples
- Tangent vectors
- Connexions
- Covariant differentiation
- Elements of general Riemannian geometry-Riemannian metric
- The fundamental theorem of local Riemannian Geometry
- Differential parameters
- Curvature tensor
- Geodesics
- Geodesics curvature
- Geometrical interpretation of the curvature tensor and special Riemannian spaces
- Contravariant and covariant vectors and tensors
- Mixed tensors
- Symmetric and skew-symmetric tensors
- Algebra of tensors
- Contraction and inner product
- Quotient theorem
- Reciprocal tensors
- Christoffel's symbols
- Covariant differentiation
- Gradient
- Divergence and curl in tensor notation.
Analysis

- A.R. Vasishtha & Others

[Fully Solved Series Available]

- Axiomatic study of real numbers
- Completeness property in R
- Archimedean property
- Countable and uncountable sets
- Neighbourhoods
- Interior points
- Limit points
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- Perfect sets
- Bolzano-Weierstrass theorem
- Sequences of real numbers
- Subsequences
- Bounded and monotonic sequences
- Convergent sequences
- Cauchy's theorems on limit
- Cauchy sequence
- Cauchy's general properties of convergence
- Sequential continuity
- Boundness and intermediate value properties of continuous functions
- Uniform continuity
- Meaning of sign of derivative
- Riemann integral
- Integrability of continuous and monotonic functions
- Fundamental theorem of integral calculus
- Mean value theorems of integral calculus
- Improper integrals and their convergence
- Comparison test
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- Dirichlet's test
- Integral as a function of a parameter and its differentiability and integrability
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- Analytic functions
- Cauchy Riemann equations (Cartesian and polar form)
- Harmonic functions
- Orthogonal system
- Power series as an analytic function
- Elementary functions
- Mapping by elementary functions
- Linear and bilinear transformations
- Fixed points
- Cross ratio
- Inverse points and critical points
- Conformal transformations.

Linear Programming

- A.R. Vasishtha & Others

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- Linear programming problems
- Statement and formation of general linear programming problems
- Graphical method
- Slack and surplus variables
- Standard and matrix forms of linear programming problem
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- Convex sets
- Fundamental theorem of linear programming
- Simplex method
- Artificial variables
- Big-M method
- Two phase method
- Resolution of degeneracy
- Revised simplex method
- Sensitivity Analysis
- Duality in linear programming problems
- Dual simplex method
- Primal-dual method
- Integer programming
- Transportation problems
- Assignment problems
- Goal Programming
- Concept of goal programming, formulation and methodology for solution of goal programming.

Mathematical Methods

- A.R. Vasishtha & Others

- Definition of a sequence
- Theorems on limits of sequences
- Bounded and Monotonic sequences
- Cauchy sequence
- Limit superior and limit inferior of a sequence
- Subsequence
- Series of non-negative terms
- Comparison tests
- Cauchy's integral test
- Ratio tests
- Root test
- Raabe's logarithmic
- De Morgan and Bertrand's tests
- Alternating series
- Leibnitz's theorem
- Absolute and conditional convergence
- The concept of transform
- Integral transform
- Kernel
- Laplace Transformation
- Linearity of the Laplace transformation
- Existence theorem for Laplace transforms
- Laplace transforms of derivatives and integrals
- Shifting theorems
- Differentiation and Integration of Laplace transforms
- Convolution theorem
- Inverse Laplace transforms
- Solution of system of differential equations using the Laplace transformation
- Fourier transforms (finite and infinite)
- Fourier integral
- Applications of Fourier transform to boundary value problems
- Fourier series
- Calculus of variations
- Variational problems with fixed boundaries
- Euler's equation for functionals containing first order derivative and one independent variable
- Extremals
- Functionals dependent on higher order derivatives
- Functionals dependent on more than one independent variable
- Variational problems in parametric form
- Invariance of Euler's equation under coordinates transformation
- Partial differential equations of the first order
- Lagrange's solution
- Some special types of equations which can be solved easily by methods other than the general methods
- Charpit's general method of solution
- Partial differential equations of the second and higher orders
- Classification of linear partial differential equations of second order
- Homogeneous and non-homogeneous equations with constant coefficients
- Partial differential equations reducible to equations with constant coefficients
- Monge's method.

Abstract Algebra

- A.R. Vasishtha & Others

- Automorphism
- Inner automorphism
- Automorphism groups and their computations
- Conjugacy relations
- Normalizer
- Counting principle and the class equation of a finite group
- Center of group of prime power order
- Sylow's theorems
- Sylow p-subgroup
- Prime and maximal ideals
- Euclidean Rings
- Principal ideal rings
- Polynomial Rings
- Polynomial over the Rational Field
- The Eisenstein Criterion
- Polynomial Rings over Commutative Rings
- Unique factorization domain
- R is unique factorization domain implies so is \( P[x_1, x_2, ..., x_n] \)
- Direct sum
- Quotient space
- Linear transformations and their representation as matrices
- The Algebra of linear transformations
- Rank nullity theorem
- Change of basis
- Linear...
Differential Geometry & Tensor Analysis

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Differential Geometry & Tensor Analysis

–A.R. Vasishtha & Others

- Local theory of curves
  - Space curves
  - Examples
  - Plane curves
  - Tangent and normal and binormal
  - Osculating plane
  - Normal plane and rectifying plane
  - Helices
  - Serret-Frenet apparatus
  - Contact between curve and surfaces, tangent surfaces
  - Involutes and evolutes of curves
  - Intrinsic equations
  - Fundamental existence theorem for space curves
  - Local theory of surfaces - Parametric patches on surface curve of a surface
  - Surfaces of revolutions
  - Helicoids
  - Metric-first fundamental form and arc length
  - Local theory of surfaces (Contd.)
  - Direction coefficients
  - Families of curves
  - Intrinsic properties
  - Geodesics
  - Canonical geodesic equations
  - Normal properties of geodesics
  - Geodesics curvature
  - Geodesics polars
  - Gauss-Bonnet theorem
  - Gaussian curvature
  - Normal curvature
  - Meusnier's theorem
  - Mean curvature
  - Gaussian curvature
  - Umbilic points
  - Lines of curvature
  - Rodrigue's formula
  - Euler's theorem
  - The fundamental equation of surface theory - The equation of Gauss
  - The equation of Weingarten
  - The Mainardi-Codazzi equation
  - Tensor algebra: Vector spaces
  - The dual spaces
  - Tensor product of vector spaces
  - Transformation formulae
  - Contraction
  - Special tensor
  - Inner product
  - Associated tensor
  - Differential Manifold-examples
  - Tangent vectors
  - Connexions
  - Covariant differentiation
  - Elements of general Riemannian geometry-Riemannian metric
  - The fundamental theorem of local Riemannian Geometry
  - Differential parameters
  - Curvature tensor
  - Geodesics
  - Geodesics curvature
  - Geometrical interpretation of the curvature tensor and special Riemannian spaces

Tensor Analysis
- Contravariant and covariant vectors and tensors
- Mixed tensors
- Symmetric and skew-symmetric tensors
- Algebra of tensors
- Contraction and inner product
- Quotient theorem
- Reciprocal tensors
- Christoffel's symbols
- Covariant differentiation
- Gradient
- Divergence and curl in tensor notation.

B.Sc. Statistics

689-08

Probability

–Dr. Arun Kumar & Dr. Alka Chaudhary

- Introduction to Probability Theory
  - History and Relevance of Probability Theory
  - Some Basic Definitions of Probability
  - Sample Space and Algebra of Events
  - Rules of Counting
  - Three Approaches to Probability
  - Probability Rules
  - Independent Events
  - Baye's Theorem
  - Random Variables and Mathematical Expectation
  - Random Variables
  - Discrete Probability Distributions
  - Continuous Probability Distributions
  - Joint Probability Distributions
  - Marginal Distributions
  - Conditional Distributions
  - Independence of Random Variables and Mathematical Expectation
  - Independence of Random Variables
  - Mathematical Expectation
  - Laws of Expectation
  - Moments
  - Correlation Coefficient of Two Random Variables
  - Conditional Expectation
  - Generating Functions and Law of Large Numbers
  - Introduction of Generating Functions
  - Chebyshev's Inequality
  - Law of Large Numbers
  - Central Limit Theorem
  - Tables.

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Probability Distribution & Numerical Analysis

–Dr. Arun Kumar & Dr. Alka Chaudhary

- Discrete Univariate Distributions
  - Introduction
  - Discrete Uniform Distribution
  - Bernoulli Distribution
  - Binomial Distribution
  - Poisson Distribution
  - Negative Binomial Distribution
  - Hypergeometric Distribution
  - Continuous Univariate Distributions
  - Introduction
  - Uniform Distribution
  - Normal Distribution
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  - Gamma Distribution
  - Beta Distribution
  - Cauchy Distribution
  - Laplace Distribution
  - Pareto Distribution
  - Exact Sampling Distributions
  - Introduction
  - Distributions of Functions of Random Variable
  - Chi-Square Distribution
  - t-Distribution
  - F-Distribution
  - Inter-Relationships between \( \chi^2, t \) and F-Distribution
  - Bivariate Normal Distribution
  - Finite Differences
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  - Symbol used in Finite Differences Calculus
  - The Operators \( E \) and \( A \)
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  - The Difference Table
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  - Differences of Zero
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  - Interpolation
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  - Interpolation with Equal Intervals
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  - Central Differences
  - Numerical Integration
  - Introduction
  - General Quadrature Formula for Equidistant
  - Contd...
...Contd: Probability Distribution & Numerical Analysis

Ordinates • The Trapezoidal Rule • Simpson’s One-Third Rule • Simpson’s Three Eighth’s Rule • Weedle’s Rule • Error in Quadrature Formula • Cote’s Method • Numerical Differentiation • Introduction • Derivatives Using Forward Difference Formula • Derivatives Using Backward Difference Formula • Derivatives Using Central Difference Formulae • Derivatives of a Function when the given Arguments are not Equally Spaced • Tables.

Probability Distribution & Theory of Attributes — Dr. Arun Kumar & Dr. Alka Chaudhary

• Discrete Univariate Distributions • Introduction • Discrete Uniform Distribution • Bernoulli Distribution • Binomial Distribution • Poisson Distribution • Negative Binomial Distribution • Hypergeometric Distribution • Continuous Univariate Distributions • Introduction • Uniform Distribution • Normal Distribution • Exponential Distribution • Gamma Distribution • Beta Distribution • Cauchy Distribution • Exact Sampling Distributions • Introduction • Distributions of Function of Random Variables • Chi-Square Distribution • t-Distribution • F-Distribution • Inter-Relationships between \( \chi^2, t \) and \( F \)-Distributions • Theory of Attributes • Introduction • Concept and Definitions • An Important Notation • Consistency of Data • Independence of Attributes • Association of Attributes • Coefficient of Association • Coefficient of Colligation • Contingency Table • Association in a Contingency Table • Tables.

Statistical Methods — Dr. Arun Kumar & Dr. Alka Chaudhary

• Definition, Functions, Limitations and Importance of Statistics • Introduction • Definition of Statistics • Other Popular Definitions of Statistics • Function of Statistics • Limitation of Statistics • Distrust of Statistics • Importance of Statistics • Statistical Tools Used in Economic Analysis • Types of Data and Scales • Introduction • Census and Sampling • Types of Data • Collection and Scrutiny of Data • Introduction • Primary and Secondary Data • Method of Collection • Scrutiny of the Data • Organisation of Data • Introduction • Classification • Object of Classification • Basis of Classification • Frequency Distribution • Method of Construction of Discrete Frequency Distribution • Method of Construction of Continuous Frequency Distribution • Basic Principles for Forming Grouped Frequency Distribution • Sturges Rule for Number of Classes and Size of Class Interval • Cumulative Frequency Distribution • Tabulation • Types of Tables • Difference between Classification and Tabulation • Diagrammatic Representation of Data • Introduction • Importance and Utility of Diagrams • Limitations of Diagrams • Rules for Constructing Diagrams • Types of Diagrams • Limitations of Diagrammatic Representation • Graphic Representation of Data • Introduction • Graphs of Frequency Distribution • Stem and Leaf Diagram • Box Plot or Box and Whisker Diagram • Measures of Central Tendency • Introduction • Objectives of Average • Characteristics of a Good Average • Various Measures of Central Tendency • Partition Values or Quantiles • Mode • Measures of Dispersion, Skewness and Kurtosis • Introduction • Objects and Importance of Dispersion • Characteristics for Satisfactory Measures of Dispersion • Absolute and Relative Measure of Variation • Measures of Dispersion • Root Mean Square Deviation • Relation between \( \sigma \) and \( S \) • Effect of Change of Origin and Scale on Standard Deviation • Combined Standard Deviation • Mathematical Properties of Standard Deviation • Moments • Relation between \( \mu \) and \( \mu' \) • Effect of Change of Origin and Scale on Moments • Sheppard’s Correction on Moments • Charlier’s Check • Pearson’s Coefficients and Fisher’s \( \gamma \) Coefficients • Skewness • Measures of Skewness • Kurtosis • Method of Least Squares and Curve Fitting • Introduction • Method of Least Squares • System of Linear Equations • Curve Fitting • Correlation and Regression • Univariate and Bivariate Distributions • Correlation • Types of Correlation • Scatter Diagram or Dot Diagram • Karl Pearson’s Coefficient of Correlation • Assumptions • Properties of Correlation Coefficient (\( r \)) • Correlation in Grouped Data • Coefficient of Determination • Rank Correlation • Regression • Angle between Two Regression Lines • Properties of Regression Coefficients • Method of Fitting Regression Lines from a Bivariate Data • Bi-spherical Correlation • Multiple and Partial Correlation • Introduction • Multiple Correlation • Partial Correlation • Multiple Regression Equation • Mathematical Notations (Yule’s Notation) • Properties of Residuals • Variance of Residuals • Expression for Coefficient of Multiple Correlation • Expression for Coefficient of Partial Correlation • Theory of Attributes • Introduction • Concepts and Definitions • An Important Notation • Consistency of Data • Independence of Attributes • Association of Attributes • Coefficient of Association • Coefficient of Colligation • Contingency Table • Association in a Contingency Table • Logarithm and Antilogarithm Table.

Statistical Inference — Dr. Arun Kumar & Dr. Alka Chaudhary

• Point Estimation • Introduction • Population and Sample • Parameter and Statistic • Theoretical Population and its Random Sample • Sampling Distribution • Standard Error • Statistical Inference: An Overview • Estimation • Criteria of a Good Estimator • Methods of Estimation • Testing of Hypothesis • Introduction • Hypothesis and its Types • Critical and Acceptance Region • Two Types of Error • Level of Significance • Power Function

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Survey Sampling

- Introduction and Basic Concepts of Sampling
- Introduction
- Advantages of Sample Survey
- Disadvantages of Sample Survey
- The Principal Steps in a Sample Survey
- Concepts, Definitions and Terminology
- Desirable Properties of an Estimator
- Sampling Errors
- Non-Sampling Errors
- Sampling Distribution
- Various Sampling Procedures
- Simple Random Sampling
- Introduction
- Various Probabilities of Selection
- How to Select a Simple Random Sample?
- Different Sets of Random Numbers
- Modified Procedure Based on Random Numbers
- Notation and Terminology
- Some Theorems Relating to Simple Random Sampling
- Without Replacement (SRSW)
- Some Theorems Relating to Simple Random Sampling
- with Replacement (SRSWR)
- Confidence Interval
- Stratified Random Sampling
- Introduction
- Advantages of Stratified Random Sampling
- Over Simple Random Sampling
- Notations and Terminology
- Two Estimates of Population Mean
- Confidence Limits
- Allocation of Sample Size
- Systematic Sampling
- Difference between Stratified Random Sampling and Systematic Sampling
- Advantages of Systematic Sampling
- Disadvantages of Systematic Sampling
- Uses of Systematic Sampling
- Notation and Terminology
- Estimator of the Population Mean
- Systematic Sampling versus Simple Random Sampling
- Comparison of Systematic with Simple and Stratified Random Sampling
- Population with Periodic Variation
- Auto-Correlated Population
- Population in Random Order
- Ratio and Regression Method of Estimation
- Introduction
- Concept of $r$ and $R$
- Notations and Terminology
- Ratio Estimator
- Bias of the Ratio Estimator
- First and Second Order Approximation to Bias
- Mean Square Error of Ratio Estimate
- Conditions for which Ratio Estimate is Better than SRS
- Properties of Ratio Estimate
- Why Ratio Estimation is Used?
- Difference Estimate
- Value of $k$ for which Variance of $r_D$ is Minimum
- Regression Estimate
- Bias of Regression Estimate
- Mean Square Error of Regression Estimate
- Some other Sampling Schemes
- Cluster Sampling
- Notations and Terminology
- Difference between Stratified Random Sampling and Cluster Sampling
- Efficiency with Respect of SRS
- Clusters of Unequal Size
- Double Sampling in Ratio Method of Estimation
- Regression Estimate in Double Sampling
- Double Sampling for Stratification
- Non-Sampling and Sampling Errors
- Introduction
- Classification of Errors
- Type of Non-Sampling Errors
- Control Measure
- Statistical Organisation in India
- Central Statistical Organisation (C.S.O)
- National Sample Survey Organisation (NSSO)
- Governing Council
- Working Groups (WG)
- Socio-Economic Surveys
- Sarvekshana, NSSO Bulletin
- United Nations World Food Programme (UNWFP)
- Agricultural Statistics
- Price Data Collection
- Urban Frame Survey
- Industrial Statistics
- Annual Survey of Industries
- Labour Bureau
- Army Statistical Organisation (ASO)
- Some Non-Government Statistical Organisations
- Statistical Organisation in States
- Statistical Organisation in U.P. and Uttaranchal
- Tables.

Analysis of Variance & Design of Experiments

- Analysis of Variance
- Introduction
- Meaning
- Assumptions
- Analysis of Variance (One-way Classified Data)
- Analysis of Two Way Classified Data with one Observation Per Cell
- Design of Experiments
- Introduction
- Meaning and Need
- Nomenclature Used in Design of Experiments
- Some Basic Points Regarding the Planning of an Experiment
- Three Principles of Design of Experiment
- Size and Shape of Plots
- Size and Shape of Blocks
- Different Experimental Designs
- Missing Plot Technique in R.B.D.
- Two Missing Observations
- Missing Plot Technique in L.S.D.
- Factorial Experiments
- Two Factors Each at 2 Levels ($2^2$ Factorial)
- Main Effects and Interactions
- Sum of Squares due to Factorial Effects
- Tests for Factorial Effects
- Yate’s Method of Computing Factorial Effects Total
- The Case of 3 Factors
- The General Case
- Confounding
- Situations where Partial Confounding is Preferable
- How to find Confounded Effects?
- Partial Confounding in a $2^3$ Experiment
- Split Plot Design
- Comparison of a Split Plot Design with R.B.D.
- Strip Plot Design
- Analysis of Covariance
- Tables.

Applied Statistics

- Time Series
- Introduction
- Definitions
- Applications of Time Series Analysis
- Components of a Time Series
- Analysis of Time Series
- Measurement of Trend
- Measurement of Seasonal Variations
- Residual Method for Isolation of Cyclic Variations
- Index Numbers
- Introduction
- Characteristics of Index Numbers
- Uses of Index Numbers
- Points to be Considered in the Construction of Index Numbers
- Types of Index Numbers

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Non-Parametric Methods & Numerical Analysis

- Non-Parametric Inference
  - Introduction
  - Advantages and Disadvantages of Non-Parametric Tests/Methods
  - The Sign Test
  - Wilcoxon Signed Rank Test
  - Mann-Whitney U Test
  - The Runs Test
  - The Median Test
  - Kolmogorov-Smirnov Test (K-S Test)
  - Spearman’s Rank Correlation Coefficient Test
- Finite Differences
  - Introduction
  - What is Quality?
  - How Quality is Measured?
- Statistical Quality Control
  - Introduction
  - What is Quality?
  - How Quality is Measured?

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  - Different Kinds of Linear Programming Problems
  - Basic Requirements of a C.P. Problem
  - Assumptions of Linear Programming
  - Applications of Linear Programming in Different Areas
  - Steps Involved in the Formulation of C.P. Problem
  - Solution of a Linear Programming Problem
  - Some Special Cases
  - Minimisation of Objective Function
  - Problem of Converting Minimise into Maximise
  - Transportation Problem
  - Introduction
  - Test for Optimality (Modi Method)
  - Unbalanced Transportation Problem
  - Degeneracy in Transportation Problem
  - Transportation Problem of Maximum Profit
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  - Unbalanced Assignment Problem
  - Max-type Assignment Problems
  - Restrictions on Assignments (Prohibitive Assignment)
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  - Characteristics of a Computer
  - History of Computers
  - The Computer Generations
  - Limitations and Applications of Computer
  - Number Systems
  - Computer Organisation
  - Input Devices
  - Output Devices
  - Memory Devices
  - Classification of Secondary Storage Devices
  - Hardware and Software
  - Operating Systems
  - Classification of Computers
- Communication and Computer Languages
  - Introduction
  - Communication
  - Digital and Analog Signals
  - Modem
  - Networking
  - Internet
  - Computer Languages
  - Introduction to Database Management System (DBMS)
- Introduction to C Language
  - Algorithm and Flow Chart
  - Introduction to C Language
  - Learning Fundamentals
  - Operators and Expressions
  - Library Functions
  - C Instructions
  - Writing a C Program
  - Input and Output Statements

Control Statements and Arrays
- Introduction
- Selection Statements
- Repetitive Iterative Statements
- Jumping Statements
- Arrays
- Functions
- Appendices.
M.Sc. Mathematics

(Book for Honours & Post-Graduate Students of All Indian Universities and Competitive Examination)

211-12

Analytical Solid Geometry
(Analytical Geometry of Three Dimensions)
—A.R. Vasishtha & D.C. Agarwal

- Central Conicoids
  - The Ellipsoid
  - The Hyperboloid of one sheet
  - The Hyperboloid of two sheets
  - The tangent plane
  - The condition of tangency
  - The Director Sphere
  - The Polar Plane
  - Properties of the polar planes and the polar lines
  - Locus of chords bisected at a given point
  - Normal to a Conicoid
  - Number of normals
  - Cubic curve through the feet of the normals
  - To find the equation of the cone through six concurrent normals (the six normals drawn from a point to an ellipsoid)
  - Diametral plane
  - Conjugate diameters and conjugate diametral planes
  - The relationship between the co-ordinates of the points P, Q, R where OP, OQ and OR are the conjugate semi-diameter of an ellipsoid
  - Properties of conjugate semi-diameters of an ellipsoid
  - The Cone
  - The Paraboloids
  - The elliptic paraboloid
  - The hyperbolic paraboloid
  - The general equation
  - The normal
  - Cubic curve through the feet of the normals
  - Generating Lines
  - The generation lines of a hyperboloid of one sheet
  - Properties of the generating lines of hyperboloid of one sheet
  - Properties of the generating lines of hyperboloid of one sheet
  - Perpendicular generators
  - The generating lines of a hyperbolic paraboloid
  - Properties of generators of a hyperboloid
  - Perpendicular generators
  - To show that the generators of the λ- and μ-systems of the hyperbolic paraboloid \(x^2/a^2 - y^2/b^2 = 2z/c\) are parallel to the planes \(x/a \pm y/b = 0\)
- The Plane Sections of Conicoids
  - Nature of a plane section
  - Lengths and direction ratios of the axes of a central section
  - Non-central Plane Section
  - To Find the Lengths and Direction Cosines of the Axes of Non-Central Plane Section of a Central Conicoid
  - Plane Sections of a Paraboloid
  - To determine the nature of a given section of a paraboloid
  - To find the lengths and direction ratios of the axes of the section of the paraboloid
- Circular Sections
  - To determine the circular sections of an ellipsoid
  - To show that any two circular sections of an ellipsoid which are not parallel lie on a sphere
  - The circular sections of any central conicoid
  - The circular sections of the paraboloid
  - Umbilics
  - Umbilics
  - Definition
  - To determine the real umbilics of the ellipsoid
  - To determine the real umbilics of the paraboloid
  - Confocal Conicoids
  - Confocal conicoids
  - Definition
  - Three confocals through a given point
  - Three paraboloids through a given point
  - To prove that one confocoid confocal with a given confocoid touches a given plane
  - To prove that the confocal confocoids cut one another at right angles at all their common points, i.e., the tangent planes at any common point are at right angles
  - Elliptic co-ordinates
  - Focal Conics
  - Properties regarding the normals to three confocal conicoids through a given point P
  - The foci of conicoids
  - Reduction of General Equation of Second Degree
  - Points of Intersection
  - The tangent plane
  - The Normal
  - The Polar Plane
  - The enveloping cone
  - The enveloping cylinder
  - To find the locus of the chords which are bisected at a given point \((\alpha, \beta, \gamma)\)
  - The diametral plane
  - Principal planes and the principal directions
  - Orthogonality of the principal directions
  - Transformation of \(f(x, y, z)\)
  - The centre of the surface \(F(x, y, z) = 0\)
  - Process of reducing a general equation to the standard form and to discuss the nature
  - Surface of Revolution
  - To find the condition that a general equation of second degree namely \(F(x, y, z) = 0\), may represent a surface of revolution.

212-21

Advanced Differential Calculus
—J. N. Sharma

- Change of Independent Variables
  - To change the independent variable into the dependent variable
  - To change the independent variable \(x\) into another variable \(z\); where \(x = \phi(z)\)
  - Differentials
  - Total and partial differential coefficients
  - Transformation in the case of two independent variables
  - Transformation from Cartesian to polar co-ordinates and vice versa
  - Orthogonal transformation of \(V^2 V\)
  - Maxima and Minima (Several Independent Variables)
  - Necessary condition for the existence of maxima or minima
  - Algebraic lemma regarding the sign of quadratic expressions
  - Lagrange’s condition for two independent variables
  - Three independent variables
  - Several independent variables
  - Lagrange’s methods of undetermined multipliers
  - Jacobians
  - Definition
  - Case of function of functions; Jacobian implicit functions
  - Necessary and sufficient condition for a Jacobian to vanish;
  - Covariants and invariants
  - Continuity and Differentiability
  - Functions, Limits, Continuity
  - The four functional limits at a point
  - Kinds of discontinuous Salts, Theorems on continuity
  - Theorems on discontinuous functions, Pointwise discontinuous function
  - Uniform continuity
  - Absolute continuity
  - Continuity of a function of more than one variable
  - Differentiability
  - Meaning of the sign of the derivative, Geometrical meaning of a derivative
  - The chain rule
  - Darboux Theorem
  - Rolle’s, Taylor’s and Allied Theorems
  - Rolle’s theorem
  - Geometrical interpretation of Rolle’s theorem
  - Lagrange’s mean value theorem
  - Cauchy’s mean value theorem
  - Taylor’s development of a function in a finite form with Lagrange’s form of remainder, Taylor’s theorem with Cauchy’s form of remainder
  - Taylor’s theorem with Schomilch and Roche’s from of remainder
  - Failure of Taylor’s and Maclaurin’s expansions in more than one variables.
Advanced Integral Calculus

- D.C. Agarwal

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- **214-42** Calculus of Finite Differences & Numerical Analysis

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**Advanced Integral Calculus**

- **Definite Integrals**
  - Definition
  - The definite integrals as the limit of a sum
  - Geometrical Interpretation
  - Properties of definite integrals
  - Method of differentiation under the sign of integration
  - Method of integration under the sign of integration
  - Principal and General Values of a definite integral

- **Euler's Integrals**
  - Definition (Beta and Gamma functions)
  - Symmetrical property of Beta function
  - To evaluate Beta function
  - To evaluate Gamma function
  - Transformation of Gamma function
  - Another form of Beta function
  - B (l, m) = \( \frac{\Gamma(l)/\Gamma(l+m)}{\Gamma(l+m)} \)
  - Other transformation of Beta functions

- **Integration of Multiple Integrals**
  - Double Integration (Cartesian co-ordinates)
  - Double Integration (Polar co-ordinates)
  - Multiple Integrals
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  - Transformation of multiple integrals
  - Transformation of implicit functions
  - Transformation of the element of a surface
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- **Integration with Unequal Intervals**
  - Lagrange's Method
  - Dirichlet's Theorem
  - Liouville's Extension of Dirichlet's Theorem
  - Volumes and Surfaces
  - Volume (Cartesian Co-ordinates)
  - Volume (Polar-Co-ordinates)
  - Area of the surface
  - Centre of Gravity
  - Moment of inertia
  - Convergence of Improper Integrals
  - Definition and kinds of improper integral
  - Convergence of improper integral of first kind
  - Necessary and sufficient condition for the convergence of \( \int_{a}^{\infty} f(x) \, dx \)
  - Tent for the convergence of \( \int_{a}^{\infty} f(x) \, dx \)

- **Calculus of Finite Differences & Numerical Analysis**

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  - Finite Differences
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  - Difference Formulae
  - Fundamental Theorem of Difference Calculus
  - The Difference Table
  - The Operator E
  - Properties of the Operators E and \( \Delta \)
  - Relation between Operator E of Finite Differences and Differential Coefficient D of Differential Calculus
  - One or More Missing Terms
  - Factorial Notation
  - Methods of Representing any Given Polynomial in Factorial Notation
  - Differences of Zero
  - Leibnitz's Rule
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  - Direct Methods (using formula)
  - Maxima and Minima of a Tabulated Function
  - Numerical Integration
  - A General Quadrature Formula for Equidistant Ordinates
  - The Trapezoidal Rule
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  - Method of Reversion of Series
  - Summation of Series
  - To Find the Sum to n Terms of a Series Whose general term is the first difference of another function
  - A Series with General Term of the Form \( u_n = \phi(x) + a_n \), where \( \phi(x) \) is Some Rational Integral Function of \( x \) of Degree \( n \)
  - Summation by Parts
  - To Prove that \( \sum_{x=1}^{n} u_x = \left[ \Delta^{-1} u_x \right]_1^{n+1} \)
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...Contd: Calculus of Finite Differences & Numerical Analysis

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**215-50**

**Differential Equations (Gen)**

- J.N. Sharma & R.K. Gupta

* Elementary Concepts * The complete solution of a differential equation of the nth order contains n-arbitrary independent constants * If y_1, y_2, ..., y_n are solutions of an equation then y = c_1y_1 + c_2y_2 + ... + c_ny_n is also a solution * Independence of constants of integration * Necc. and Suff. cond. for parameters c_1 and c_2 to be independent * Linear dependence and independence of solutions of equations * Necc. and Suff. cond. for n solutions to form a system of linearly independent integrals * Linear Equations of Second Order * Complete solution in terms of a known integral * To find particular integral of d^2y/dx^2 + P dy/dx + Qy = 0 * Removal of the first derivative * Transformation of the equation by changing the independent variable * Method of Variation of parameters * Methods of Operational Factors * Ordinary Simultaneous Differential Equations * Simultaneous linear diff. equations with constant coefficient * Simultaneous equations in a different form * Solution of simultaneous equations of the form dx/P = dy/Q = dz/R • Total Differential Equations (Pfaffian Differential Forms and Equations) • Pfaffian Differential Equations * Total Differential Equation for Pfaffian Differential Equation in three variables * Necc. and Suff. condition for integrability of single diff. equation P dx + Q dy + R dz = 0 * The condition for exactness • Methods for solving P dx + Q dy + R dz = 0 • Solution of P dx + Q dy + R dz = 0, when it is exact and homogeneous of degree n - 1 • Geometrical interpretation of the equation P dx + Q dy + R dz = 0. The locus of P dx + Q dy + R dz = 0 is orthogonal to the locus of dx/P = dy/Q = dz/R • The non-integrable single equation • Equations containing more than three variables • General method of solution of the equations containing more than three variables • Integration in Series • General method of solving a diff. eqn. • Case I. Roots of indicial equation equal • Case II. Roots of indicial equation, unequal and differing by quantity not an integer • Case III. Roots of indicial equation differing by an integer, making a coefficient of y derivative • Some cases where the methods fails • Series solution about a particular point • The particular Integral • Method of differentiation •

Contd...
...Contd: Differential Equations (Gen)

Picard’s Iteration Methods, Uniqueness and Existence Theorem • Picard’s Iteration Method • Existence and uniqueness of solutions • The Lipschitz condition • Existence theorem • Uniqueness theorem • Existence and uniqueness theorem • Theorem • Partial Differential Equations of the First Order • Derivative of partial differential equation • Definitions • Linear partial differential equation of order one • Lagrange’s Linear equation • Lagrange’s solution of the linear equation • Geometrical Interpretation of Lagrange’s linear equation • The linear equation with n independent variables • Special types of equations • Standard I. Equation of the form \( f(p, q) = 0 \) • Standard II. Equation of the form \( f(x, y) = 0 \) • Standard III. Equations of the form \( f(x, y) = f(y, p) \) • Standard IV. Equations of the form \( x = px + qy + f(p, q) \) • General Method of solution • Two independent Variables Charpit’s Method • Three or more independent Variables Jacobbi’s Methods • Partial Differential Equations with Constant Coefficients • Homogeneous linear equations with constant coefficients • Solution of the linear partial differential equations • To find the complementary function • When the auxiliary equation has equal (repeated) roots • The particular integral • Short Methods • Exceptional case when \( f(a, b) = 0 \) • General Methods • Non-homogeneous Linear equations with constant coefficients • Particular Integral • Equation reducible to homogeneous linear form • Partial Differential Equations of the Second Order • Monge’s Methods • Monge’s method of integrating \( Rr + Ss + Tt = V \) • Monge’s method of integrating \( Rr + Ss + Tt = V \) • Classification of Linear Partial Differential Equations • Classification of linear partial differential Equations of second order • Homogeneous Linear Equations with Variable Coefficients • Homogeneous linear Equations • Methods of Solution • Equations reducible to homogeneous form • Singular Solution • Discriminant • Extraneous Roots • Exact Differential Equations and Equations of other Particular Forms • Exact Diff. Eqn. (Definition) • Condition of exactness of a linear equation of order \( n \) • Integrating factor • Non-linear equation • An equation which does not contain \( y \) directly • An equation which does not contain \( x \) directly • An equation of the form \( \frac{d^n y}{dx^n} = f(x) \) • An equation of the form \( \frac{d^n y}{dx^n} = f(y) \) • An equation of the form \( \frac{d^n y}{dx^n} = f(x, y) \)

\[
\left( \frac{d^n y}{dx^n} \right)^2 + \left( \frac{d^{n-2} y}{dx^{n-2}} \right) x = 0
\]

• Equation in which order of the differential coefficients differ by unity • Numerical Integration • Simpson’s Rule • Numerical approximation • Legendre Polynomials • Legendre’s equation • Solution of Legendre’s Equation • Definition of \( P_n(x) \) and \( Q_n(x) \) • General solution of Legendre’s equation • To show that \( P_n(x) \) is the coefficient of \( x^n \) in the expansion in ascending powers of \( h \) of \( (1 - 2hx + h^2)^{-1/2} \) • Laplace’s Definite Integrals for \( P_n(x) \) • Orthogonal properties of Legendre’s Polynomials • Recurrence formulae • Beltrami’s Result • Christoffel’s Expansion • Christoffel’s Summation Formulae • Rodrigues Formulae • Some Bounds on \( P_n(x) \) • Even and odd functions • Expansions of \( x^n \) in Legendre’s Polynomials • General Results • An important Case • Trigonometrical series for \( P_n(x) \) • Legendre’s Function of the Second Kind \( Q_n(x) \) • Legendre’s functions of the Second Kind • Neumann’s Integral • Recurrence formulae for \( Q_n(x) \) • Relation between \( P_n(x) \) and \( Q_n(x) \) • Christoffel’s Second Summation formula • Complete solution of Legendre’s equation (other form) • Bessel Functions • Bessel’s equation (Def.) • Solution of Bessel’s General Differential Equations • General solution of Bessel’s equation • Integration of Bessel’s equation in series for \( n = 0 \) • Definition of \( J_n(x) \) • Recurrence formulae for \( J_n(x) \) • Generating function for \( J_n(x) \) • Some Trigonometric expansion involving Bessel’s functions • A second solution of Bessel’s Equation • Hermite Polynomials • Hermite Differential Equation • Solution of Hermite Equation • Hermite’s Polynomials • Generating function • Other forms for Hermite Polynomials • To find first few Hermite Polynomials • Orthogonal properties of Hermite polynomials • Recc. Formula for Hermite Polynomials • Laguerre Polynomials • Laguerre’s Differential Equation • Solution of Laguerre Equation • Laguerre Polynomials • Generating function • Rodrigues formula • To find first few Laguerre Polynomials • Orthogonal Prop. of Laguerre Polynomials • Rec. formula for Laguerre Polynomials • Chebyshev Polynomials • Chebyshev’s Equation • Chebyshev’s Polynomials • To prove that \( T_n(x) \), \( U_n(x) \) are independent solutions of Chebyshev Equation • Important Relations for \( T_n(x) \) and \( U_n(x) \) • To find first few Chebyshev Polynomials • Generating function • Orthogonal properties of Chebyshev’s polynomials • Rec. formula for \( T_n(x) \) and \( U_n(x) \).
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Roots of Indicial Equation Unequal Differing by an Integer, One Root Making a Coefficient of y Infinity (Frobenius Method) • Roots of Indicial Equation Equal [Frobenius Method] • Solution of Legendre’s Equation (In Decending Powers of x) • The Particular Integral • Some Cases where the Frobenius Method Fails • Beta and Gamma Functions • Euler's Integrals • Elementary Properties of Gamma Function • To Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ • Transformation of Gamma Function • Symmetric Property of Beta Function i.e., $B(m, n) = B(n, m)$ • Transformation of Beta Function • Relation between Beta and Gamma Functions $B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m + n)}$, $m > 0, n > 0$ • $\int_0^{\pi/2} \sin^n \theta \cos^m \theta \, d\theta = \frac{\Gamma\left(\frac{m + 1}{2}\right) \Gamma\left(\frac{n + 1}{2}\right)}{\Gamma\left(\frac{p + q + 2}{2}\right)}$ • Legendre Duplication Formula • To Prove that $\Gamma\left(\frac{1}{n}\right) \frac{\Gamma\left(\frac{2}{n}\right)}{\Gamma\left(\frac{3}{n}\right)} \cdots \frac{\Gamma\left(n - 1\right)}{\Gamma\left(n\right)} = \frac{(2\pi)^{n - 1/2}}{\sqrt{n}}$, where $n$ is a positive integer • **Gauss Hypergeometric Equation** • The Pochhammer Symbol (Def.) • Identities Satisfied by Pochhammer Symbol $(\alpha)_n$ • Hypergeometric Series • Hypergeometric Function • Different Forms of Hypergeometric Function • Confluent Hypergeometric Function (Kummer Function) • General Hypergeometric Function • Symmetric Property of Hypergeometric Function • Particular Cases of Hypergeometric Series • Gauss's Hypergeometric Equation or Gauss’s Equation or Hypergeometric Equation • Solution of the Hypergeometric Equation • Derivatives of Hypergeometric Function • $n$th Derivative of Hypergeometric Function • Derivatives of Hypergeometric Function at $x = 0$ • Integral Formula for Hypergeometric Function • Kummer’s Theorem (For Hypergeometric Function) • Gauss’s Theorem • Vandermonde’s Theorem • Confluent Hypergeometric Equation (Or Kummer’s Equation) and its Solution • Series Solution of Confluent Hypergeometric Differential Equation near $x = 0$ when $\gamma$ is not an Integer • Derivatives of Confluent Hypergeometric Function • Integral Formula for Confluent Hypergeometric Function $\Gamma_{1,1}(\alpha, \beta; x)$ • Kummer’s Theorem (For Confluent Hypergeometric Function) • Whittaker’s Confluent Hypergeometric Function • Contiguous Hypergeometric Functions • Theorem: Contiguity Relationship • **Hermite Polynomials** • Hermite Differential Equation • Solution of Hermite Differential Equation • Hermite Polynomial Generating Function for Hermite Polynomial $H_n(x)$ • Other Forms for the Hermite Polynomials • To Find First Few Hermite Polynomials • Orthogonal Properties of Hermite Polynomials • Recurrence Formulae for Hermite Polynomials • **Partial Differential Equations of the First Order (Origin of First Order Partial Differential Equations and Classification)** • Order and Degree of a Partial Differential Equation • Classification of First Order Partial Differential Equations into Linear, Semi-linear, Quasi-linear and Non-linear • Origin (Derivation) of First Order Partial Differential Equation • Some Definitions • Lagrange’s Linear Partial Differential Equation • Lagrange’s Solution of the Lagrange’s Linear Equation (Lagrange’s Method of Solving the Linear Partial Differential Equation of Order One Namely $P\partial_x + Q\partial_y = R$) • Working Method • The Linear Partial Differential Equation with n Independent Variables • Integral Surfaces Passing Through a Given Curve • Surface Orthogonal to a Given System of Surface • Compatible System of First Order Equations • **Non-linear Partial Differential Equations of First Order (Charpit’s and Jacobis Methods)** • Solution of Partial Differential Equations of First Order and any Degree in Some Standard Forms • Standard Form I: Equation Involving Only $p$ and $q$ and no $x, y$ and $z$ • Standard Form II: Equations Involving Only $p, q$ & $z$ • Standard Form III: Equations of the Form $f_1(x, p) = f_2(y, q)$ • Standard Form IV: Equations of the Form $x = px + qy + f(p, q)$ • Charpit’s Method: General Method of Solution of Non-linear Partial Differential Equation of Order One with Two Independent Variables • Jacobis Methods • Jacobis Method of Solving a Non-linear First Order Partial Differential Equation in Two Independent Variables • **Partial Differential Equations of the Second Order with Variable Coefficients (Origin and Classification)** • Origin (Derivation) of Second Order Partial Differential Equation • Special Types of Second Order Partial Differential Equations • Solutions of Equations under Given Conditions • Classification of Linear Partial Differential Equations of Second Order in n-Independent Variables • Classification of Linear Partial Differential Equation of Second Order in Two Independent Variables • **Linear Partial Differential Equations with Constant Coefficients** • Homogeneous and Non-homogeneous Linear Partial Differential Equations with Constant Coefficients • Solution of a Homogeneous Linear Partial Differential Equation with Constant Coefficients • Methods of Finding the Complementary Function (C.F.) of the Homogeneous Linear Partial Differential Equation with Constant Coefficients • Working Method of Finding C.F. of a Homogeneous Linear Partial Differential Equation with Constant Coefficients • Determination of the Particular Integral (P.I.) of a Homogeneous Linear Partial Differential Equation with Constant Coefficients • Short Method to Find P.I. when $\phi(x, y)$ is a Function of the form $x^m y^n$ or a Rational Integral Algebraic Function of $x$ and $y$ • Short Methods of Finding P.I. When $\phi(x, y)$ is a Function of $ax + by$ • General Method of Finding the P.I. of Homogeneous Linear Differential Equation with Constant Coefficients • Non-homogeneous Linear Differential Equations with Constant Coefficients • Methods of Finding the Complementary Function (C.F.) of Reducible Non-homogeneous Linear Partial Differential Equation with Constant Coefficients • Working Method of Finding C.F. of Reducible Non-homogeneous Linear Partial Differential Equation with Constant Coefficients • Determination of the Particular Integral (P.I.) of Non-homogeneous Linear Partial Differential Equation (Reducible or Irreducible) with Constant Coefficients • Solution of Linear Partial Differential Equation with Constant Coefficients under given Geometrical Conditions • **Reduction of Second Order Partial Differential Equation into Canonical Forms (Non-linear Equations of Second Order)** • Laplace Transformation (Canonical Forms) • Working Method of Reducing a Hyperbolic Equation to Canonical Form • Working Method of Reducing a Parabolic Equation to Canonical Form • Working Method of Reducing Elliptic Equation to Canonical Form • Wave Equations (By Method of Separation of Variables) • Wave Equation • Solution of One Dimensional Wave Equation by Using the Method of Separation of Variables • Solution of One Dimensional Wave Equation Under the Given Conditions • Some Important and Useful Differential Equations and Their Solutions • Solution of Two Dimensional Partial Differential Equations.
Wave Equation by the Method of Separation of Variables • Vibration of a Circular Membrane (Solution of Two Dimensional Wave Equation in Polar Coordinates) • Solution of Three Dimensional Wave Equation by the Method of Separation of Variables • Wave Equation in Cylindrical Coordinates • Solution of Wave Equation is Cylindrical Coordinates by the Method of Separation of Variables • Wave Equation in Spherical Coordinates • Solution of Wave Equation in Spherical Coordinates by the Method of Separation of Variables • Heat and Diffusion Equations (By Method of Separation of Variables) • One Dimensional Heat Equation • Heat Equation • Diffusion Equation • Solution of One Dimensional Heat Equation by Separation of Variables • Solution of One Dimensional Heat Equation under given Boundary Conditions • Solution of Two Dimensional Heat Equation in Cartesian Coordinates • Heat Equation in Plane Polar Coordinates • Solution of Heat Equation in Plane Polar Coordinates by Separation of Variables • Solution of Three Dimensional Heat Equation by the Method of Separation of Variables • Heat (Diffusion) Equation in Cylindrical Coordinates • Solution of Heat (Diffusion) Equation in Cylindrical Coordinates by the Method of Separation of Variables • Heat (Diffusion) Equation in Spherical Polar Coordinates • Solution of Heat (Diffusion) Equation in Spherical Polar Coordinates by the Method of Separation of Variables • Laplace Equations (By Method of Separation of Variables) • Laplace Equation • Solution of Two Dimensional Laplace’s (Harmonic) Equation by Using the Method of Separation of Variables • Solution of Two Dimensional Laplace’s Equation under the Given Conditions • Laplace Equation in Plane Polar Coordinates • Solution of Laplace Equation in Plane Polar Coordinates by Separation of Variables • Solution of Laplace’s Equation in Rectangular Cartesian Coordinates (x, y, z) by the Method of Separation of Variables • Laplace Equation in Cylindrical Coordinates • Solution of Laplace’s Equation in Cylindrical Coordinates by the Method of Separation of Variables • Laplace Equation in Spherical Coordinates • Solution of Laplace’s Equation in Spherical Coordinates by the Method of Separation of Variables.

Differential Geometry

216-40

Curves in Space (R^3) • Space curves • Path • Arc length • Tangent Line • Contract of nth order of a curve and surface • The osculating plane (or plane of curvature) • Tangent plane at any point of the surface f(x, y, z) = 0 • To find the osculating plane at a point of a space curve given by the intersection of the surface f(r) = 0, ψ(r) = 0 • The Principal normal and binormal • Definitions of curvature, Torsion and screw curvature • To find curvature and Torsion of curve • Helices • Intrinsic Equations (or Natural Equations) Fundamental Theorems for space curves • The circle of curvature • The osculating sphere (or sphere of curvature) • Behaviour of curve in the Neighbourhood of a point • Involute and Evolute • The spherical indicatrix or spherical images • Bertrand curves • Concept of a Surface and Fundamental Forms • Concept and Definition of a surface • Curvilinear equations of the curve on the surface • Parametric curves • Tangent plane and normal • Fundamental Forms • Two fundamental forms • First Fundamental form or Metric • Second Fundamental Form • Some Important Products • Derivatives of N, Weingarten Equations • Angle between parametric curves • Direction Coefficients • Angle between any two intersecting curves on the surface • Families of curves • Orthogonal Trajectories • Double Family of curves • Local Non-intrinsic Properties of a Surface, Curve on a Surface • Curvature of normal section • Principle Directions and Principal curvatures • Line of curvature • General surface of revolution • Joachimsthal’s Theorem • Dupin’s Indicatrix • Third Fundamental Form • Envelope, Edge of Regression and Developable • Envelope of system of surfaces whose equations involves two parameters • Ruled Surfaces (Developable and Skew) • Developable surface • Developables associated with space curves K = 0; for a developable surface • Monge’s Theorem • Conjugate directions • Asymptotic Lines • Fundamental coefficients and gaussian curvature for a ruled surface • The fundamental Equations of Surface Theory Gauss’s Formulae • The Fundamental Equations of Surface Theory (Tensur notation) • Parallel Surfaces • Whole curvature • Geodesic and Mapping of Surfaces • Geodesics • Differential equation of geodesics • Normal Property of geodesics • Geodesic curvature • Gauss Bonnet Theorem • Torsion of a geodesic • Bonnet’s theorem in relation to geodesics • Geodesics on F(x, y, z) = 0 • Geodesics parallel • Mapping of Surfaces • Some Definitions • Isometric lines and Isometric correspondence • Minding Theorem • Conformal mapping • Geodesic mapping • Tissot’s theorem • Dini’s theorem • Symbols and Abbreviations.

Dynamics of a Particle

217-16

Central Orbits • Central Forces, Elliptic, Hyperbolic and Parabolic Orbits, Apses and apsidal distances • Planetary Motion • Motion under inverse square law, Planetary Motion, Kepler’s Laws, Perihelion and Aphelion Points, Distributed Elliptic Motion • Anomalies, Planetary Motion (Continued) • Lambert’s Theorem • Tangential and Normal Acceleration, Conservation of Energy, Simple Pendulum and Constrained Motion • Motion in a smooth vertical circle, Motion on a smooth plane curve, Motion on a general curve, Motion on a circle, Elastic string • Motion on a smooth cycloid, Motion on a rough cycloid • Motion in a Resisting Medium and Motion when Mass Varies • Motion in a Straight Line in a Resisting Medium, Motion of Projectiles in a Resisting Medium • Revolving Curves • Moment of Inertia • D’Alembert’s Principle and Motion about a Fixed Axis.
Due to an oscillating flat plate distribution, vorticity alternate proof

\[
\text{Fluid Dynamics}
\]

- Basic Concepts
- Types of fluid
- Fluid properties
- Density
- Specific weight
- Specific volume
- Specific gravity
- Pressure
- Viscosity
- Temperature
- Thermal conductivity
- Specific heat
- Surface tension
- Vapour pressure
- Bulk modulus of Elasticity
- Kinematics of the Flow Field
- Lagrangian method
- Eulerian method
- Relationship between the Lagrangian and Eulerian method
- Velocity of a fluid particle at a point
- Local, convective and material derivatives
- Equation of continuity
- Equation of continuity (stream tube concept)
- Equation of continuity (cartesian coordinates)
- Equation of continuity (spherical polar coordinates)
- Equation of continuity (cylindrical polar coordinates)
- Equation of continuity (spherical coordinates)
- Equation of continuity (Lagrangian method)
- Equivalence of the two forms of the equation of continuity
- Velocity potential, irrotational flow
- Rotational flow
- Vorticity
- Vorticity vector, Vortex lines, Vortex tube
- Vortex filament
- Boundary Surface
- Conservation of Momentum
- Euler's equation of motion along a streamline
- Equation of motion of an inviscid fluid
- Equation of motion of an inviscid fluid (cartesian coordinates)
- Cauchy's integral
- Bernoulli's equation (Stream tube method)
- Conservative field of force
- Integration of Euler's equation
- Helmholtz equations
- Symmetrical forms of the equation of continuity
- Spherical symmetry
- Cylindrical symmetry
- Impulsive motion of a fluid
- Impulsive motion of a fluid (Cartesian coordinates)
- Energy equation
- Applications of Bernoulli's Theorem
- Flow over a protuberance in a closed channel
- Pitot tube
- Venturi tube
- Orifice plate
- Weirs
- Irrotational Motion
- General motion of a fluid element
- Motion of a fluid element (cartesian coordinates)
- Vorticity
- Body forces and surface forces
- Flow and circulation
- Stokes' theorem
- Kelvin's circulation theorem
- Connectivity
- Cyclic constants
- Irrotational motion in multiply-connected space
- Acyclic and Cyclic motion
- Green's theorem
- Deductions from Green's theorem
- Mean value of the velocity potential over a spherical surface
- Motion regarded as due to Sources and Sinks
- Liquid extending to infinity
- Kelvin's minimum energy theorem
- Motion in Two Dimensions
- Stream function (Plane polar coordinates)
- Physical interpretation of Stream function
- Complex potential and complex velocity
- Uniform flows
- Two dimensional Source and Sink
- Strength
- Complex potential of a source
- Two-dimensional doublet
- Complex potential of a doublet
- Images in two-dimension
- Image of a source with regard to a plane
- Image of a doublet with regard to a plane
- The circle theorem
- Image of a Source with regard to a circle
- Image of a doublet with regard to a circle
- Conformal representation
- Application to Fluid Dynamics
- General motion of a cylinder in two-dimensions
- Motion of a circular cylinder in a uniform stream
- Liquid Streaming past a fixed circular cylinder
- Two co-axial cylinders (Problem of initial motion)
- Circulation about a circular cylinder
- Blasius's theorem
- Streaming and Circulation for a fixed circular cylinder
- Equation of motion of a circular cylinder with circulation
- Elliptic coordinates
- Motion of an elliptic cylinder
- Streaming past a fixed elliptic cylinder
- Elliptic cylinder rotating in an infinite mass of liquid at rest at infinity
- Kinetic energy of rotating elliptic cylinder
- Kinetic energy when the liquid contained in a rotating elliptic cylinder
- Motion of a parabolic cylinder
- Velocity potential and stream function for a liquid streaming past a fixed parabolic cylinder
- The aerofoil
- Joukowski transformation
- Kutta-Joukowski's theorem
- D'Alambert's paradox
- Schwarz-Christoffel theorem
- Transformation of a semi-infinite strip
- Semi-infinite strip
- Infinite strip
- Flow into a Channel through a narrow slit in a wall
- Flow past a step in a deep stream
- Flow past a step in a channel
- Vortex Motion
- Properties of the vortex
- Strength of the vortex
- Rectilinear Vortices
- Velocity components
- Centre of vortices
- A case of two vortex filaments
- Stream function when the strength of the vortex filaments are equal
- Vortex pair
- Vortex doublet
- Vortex inside an infinite circular cylinder
- Vortex outside a circular cylinder
- An infinite single row of parallel rectilinear vortices of the same strength
- Two infinite rows of parallel rectilinear vortices
- Karman's vortex street
- Kirchoff vortex theorem
- Rectilinear vortex with circular section
- Rankine's combined vortex
- Rectilinear vortices with elliptic section
- Vortex sheets
- Routh theorem
- Waves
- Wave motion
- Mathematical representation of wave motion
- Standing or Stationary waves
- Classification of waves
- Surface waves
- Progressive waves on the surface of a canal
- Waves on a deep canal
- Energy of progressive wave
- Progressive waves reduced to a steady motion
- Standing or Stationary waves
- Energy of Stationary waves
- Waves at the common surface of two liquids
- Waves at an interface with upper surface free
- Group velocity
- Rate of transmission of energy
- Long waves
- Energy of a long wave
- Long waves at the common surface of two liquids bounded above and below by two fixed horizontal planes
- Irrotational Motion in Three Dimensions
- Butler's sphere theorem
- Solution of Laplace equation
- Motion of a sphere in an infinite mass of liquid at rest at infinity
- Ideal flow round a sphere
- Liquid streaming past a fixed sphere
- Concentric spheres
- Equation of motion of a sphere
- Three dimensional source and sink
- Three dimensional doublet
- Image of a source with regard to a sphere
- Motion of a liquid inside a rotating ellipsoidal shell
- Motion of an ellipsoid in an infinite mass of liquid
- Values of Stoke's stream function
- Values of Stoke's stream function
- A simple source on the X-axis
- A uniform line source along the axis
- A doublet along the axis
- Solid of revolution along their axes in an infinite mass of liquid
- Viscous Fluid Flow
- Stress analysis at a point
- State of a stress at a point
- Symmetry of stress tensor
- Alternative proof
- Stress in a fluid at rest
- Stress in a fluid in motion
- Transformation of stress-components
- Tensor character of stress matrix
- Stress quadratic
- Orthogonal principal directions
- Principal stresses and Principal directions
- Strain analysis
- Rate of Strain quadratic
- Alternative proof
- Transformation of the rates of strain
- Relation between stress and rate of strain
- Navier-Stokes equation of motion of a viscous fluid (Cartesian coordinates)
- Limitations of the Navier-Stokes equation
- Equation of energy
- Dissipation of energy
- Vorticity and Circulation in viscous fluids
- Diffusion of vorticity
- The equation of state
- Dimensional Analysis
- Reynolds number
- Buckingham's theorem
- Similitude
- Froude number
- Pressure coefficient (Euler’s number)
- Match number
- Reynolds number
- Grashof number
- Prandtl Number
- Peclet number
- Exact Solutions of the Navier-Stoke's Equation
- Laminar flow through parallel plates
- Plane couette flow
- Generalised plane couette flow
- Plane Poiseuille flow
- Flow between parallel plates (Temperature distribution)
- Plane couette flow
- Generalised Plane couette flow
- Plane Poiseuille flow
- Hagen-Poiseuille flow
- Flow through a circular pipe
- Steady flow between co-axial circular pipes
- Steady flow in pipes of elliptic cross-section
- Steady flow in pipes of equaliteral triangular section
- Steady flow in pipes of rectangular sections
- Hagen-Poiseuille flow in a circular pipe (temperature distribution)
- Laminar flow between concentric rotating cylinders
- Temperature distribution
- Steady motion of a viscous fluid due to a slowly rotating sphere
- Flow in convergent and divergent channels
- Unsteady motion of a flat plate
- Flow due to an oscillating flat plate
- Pulsatile flow between parallel surfaces
- Unsteady flow of viscous incompressible fluid between two parallel plates
- Diffusion of a...
vortex filament • Low Reynolds number solution • Solution of the Navier-Stokes equation at low Reynolds number • Slow flow past a sphere • Flow past a sphere (Aliter) • Flow past a circular cylinder • Laminar Boundary-Layer Flow • Two dimensional boundary layer equations for flow over a plane wall • Boundary layer flow along a flat plate • Boundary layer thickness • Properties of the boundary layer equations • Boundary layer flow past a wedge • Potential flow past a wedge • Potential flow around a corner • Flow in a convergent channel • Momentum integral equation for the boundary layer • Momentum and energy integral equation for the boundary layer (Aliter) • Application of the Integral Equation to boundary layers Von Karman’s Pohlhausen method • Discontinuous Motion • Properties of the free stream lines • Flow in jets and currents • Motion of two impinging jets • Direct impact of four equal jets • Borda’s Mouthpiece • Jet of a liquid through a slit • Impact of a stream on a lamina • Lubrication Theory • The generalised Reynolds equation • Flow between parallel walls • The Real bearing • One-dimensional journal bearings (Ininitely long bearing) • Ininitely short bearing • One-dimensional Thrust bearing • Step bearing • Appendix: Orthogonal curvilinear coordinate.
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Transformations ● Elliptic, Hyperbolic and Parabolic Transformations ● Some Special Bilinear Transformations ● More about Conformal Mappings ● The Transformation \( w = z^n \) ● The Transformation \( w = z^2 \) ● The Inverse Transformation \( z = \sqrt{w} \) ● The Exponential Transformation \( w = e^z \) ● The Logarithmic Transformation \( w = \log z \) ● The Trigonometrical Transformations ● The Transformation \( w = \tan^2 \left( \frac{\pi}{4} z \right) \) ● The Transformation \( w = \frac{1}{2} \left( z + \frac{1}{z} \right) \) ● Some General Techniques of Conformal Mapping ● Complex Integration ● Complex Line Integrals ● Reduction of Complex Integrals to Real Integrals ● Some Properties of Complex Integrals ● An Estimation of a Complex Integral ● Line Integrals as Functions of Arcs ● Cauchy’s Fundamental Theorem ● Second Proof of Cauchy-Goursat Theorem ● A Third proof of Cauchy-Goursat Theorem ● Cauchy’s Integral Formula ● Poisson’s Integral Formula of a Circle ● Derivative of an Analytic Function ● Higher Order Derivatives ● Morera’s Theorem ● Indefinite Integrals or Primitives ● Cauchy’s Inequality ● Liouville’s Theorem ● Expansion of Analytic Functions as Power Series : Taylor and Laurent’s Theorems ● The Zeros of an Analytic Function ● Different Types of Singularities ● Some Theorems on Poles and Other Singularities ● The Point at Infinity ● Characterization of Rotational Functions ● Maximum Modulus Principle ● The Excess of Number of Zeros Over Number of Poles of a Meromorphic Function ● Rouche’s Theorem ● Schwarz Lemma ● Inverse Function Theorem ● Fundamental Theorem of Algebra ● Analytic Continuation ● Power Series Method of Analytic Continuation ● Schwartz’s Reflection Principle ● Calculus of Residues ● Residue at Simple Pole ● Residue at a Pole of Order Greater than Unity ● Residue at Infinity ● Cauchy’s Residue Theorem ● Evaluation of Definite Integrals ● Integration Round the Unit Circle ● Evaluation of the Integrals \( \int_{-\infty}^{\infty} f(x) \, dx \) ● Jordan’s Inequality ● Jordan’s Lemma ● Evaluation of the Integrals of the form \( \int_{-\infty}^{\infty} \frac{P(x)}{Q(x)} \sin mx \, dx \) etc. ● Case of the Poles on the Real Axis ● Integrals of may Values

Functions ● Rectangular and Other Contours ● Expansion of Meormorphic Functions ● Uniform Convergence and Infinite Products ● Uniform Convergence of a Sequence ● General Principle of Uniform Convergence ● Uniform Convergence of a Series ● Weierstrass’s M-test ● Hardy’s Test ● Continuity of the Sum Function ● Term by Term Integration ● Analyticity of the Sum Function of a Series, Term by Term Differentiation ● Hurwitz Theorem ● Uniform Convergence of Power Series ● A note on Absolute and Uniform Convergence ● Infinite Products ● Three Important Theorems on Infinite Products ● The Absolute Convergence of Infinite Products ● Uniform Convergence of Infinite Products ● Entire Functions ● Mittag Leffler’s Theorem ● The Weierstrass Factorization Theorem ● Canonical Products ● The Jenson and Poisson-Jenson Formulas ● Growth, Order and Convergence Exponents of Entire Functions ● Hadmard’s Factorization Theorem ● The Gamma Function.

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Complex Analysis

--A.R. Vasishtha, Vipin Vasishtha & A.K. Vasishtha

● Complex Numbers and their Geometrical Representation ● Complex Numbers ● Properties of the Addition Of Complex Numbers ● Properties Of The Multiplication Of Complex Numbers ● Difference Of Two Complex numbers ● Division In C ● Modulus Of A Complex Number ● Conjugate Of A Complex Number ● Modulus-argument Form Or Polar Standard Form Or Trigonometric Form Of A Complex Number ● The Geometrical Representation Of Complex Numbers ● The Points On The Argand Plane Representing The Sum, Difference, Product And Division Of Two Complex Numbers ● More Properties Of Moduli And Arguments ● Theorem: The Order Relations Greater Than Or Less Than Do Not Apply To Complex Numbers ● Some Important Results About Complex Numbers ● Integral And Rational Powers Of A Complex Number ● Geometrical Applications Of Complex Number ● Complex Equation Of A Straight Line In The Complex Plane ● Equation Of A Circle In The Complex Plane ● The Spherical Representation Of Complex Numbers And Stereographic Projection ● Analytic Functions ● Curves In The Argand Plane ● Functions Of A Complex Variable ● Neighbourhood Of A Point ● Limits And Continuity ● Differentiability ● Analytic, Holomorphic And Regular Functions ● The Necessary And Sufficient Conditions For \( f(z) \) To Be Analytic ● Polar Form Of Cauchy-Riemann Equations ● Derivative of \( w = f(z) \) In Polar Form ● Orthogonal System ● Harmonic Function ● Methods Of Constructing A Regular function (Milne-Thomson’s Method) ● Multiple Valued Functions ● Conformal Mappings ● Mappings Or Transformations ● Jacobian Of A Transformation ● Conformal Mapping ● Necessary Conditions for \( w = f(z) \) To Represent A Conformal Mapping ● Sufficient Conditions For \( w = f(z) \) To Represent A Conformal Mapping ● Superficial Magnification ● The Circle ● Inverse Points With Respect To A Circle ● Some Elementary Transformations ● Linear Transformation ● Bilinear Or Linear Fractional Transformation ● Critical Points ● Resultant Or Product Of Two Bilinear Transformations ● Bilinear Transformation As The Resultant Of Elementary Bilinear Transformations With Simple Geometric Properties ● Bilinear Transformation As The Resultant Of An Even Number Of Inversions ● The Linear Group ● Equation Of A Circle Through Three Given Points ● Cross Ratio ● Preservance Of Cross-Ratio Under Bilinear Transformation ● To Find The Bilinear Transformation Which Transforms Three Distinct Points \( z_1, z_2, z_3 \) Of z-Plane Respectively Into Three Specified Points \( w_1, w_2, w_3 \) Of w-Plane ● Two Important Families Of Circles ● Preservance Of the Family Of Circles And Straight Lines Under Bilinear Transformations ● Fixed Points Or Invariant Points Of A Bilinear Transformation ● Normal Form Of A Bilinear Transformation ● Elliptic, Hyperbolic And Parabolic Transformations ● Some Special Bilinear Transformations ● Taylor’s Series ● More about Conformal Mappings (Some Special Transformations) ● The Transformation \( w = Z^n \) (Where n Is A Positive Integer) ● The Transformation \( w = z^2 \) ● The Inverse Transformation \( z = \sqrt{w} \) ●
...Contd: Complex Analysis


Hydrodynamics

- Shanti Swarup

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Infinite Series & Products

- Sequences and Limits
- Sequences of real numbers
- Bounded sequences
- Monotonic sequences
- Series
- Limits of a sequence
- Convergence of a sequence
- Cauchy’s general Principle of Convergence of a Series
- Upper and Lower bounds and limits
- Convergence of monotonic sequences
- Theorems on limits
- Cauchy’s first theorem on limits
- Cauchy’s second theorem on limits
- Theorems on limits of quotients
- Hardy’s test
- Cesàro summability
- Abel’s test
- Dirichlet’s test
- Raabe’s test
- de Moivre’s test
- De Morgan’s test
- Bertrand’s test
- Absolute and Conditional Convergence
- General Principle of Convergence
- Alternating Series
- Absolute and non-negative convergence
- Rearrangement of terms of an absolutely convergent series
- Inversion of Parentheses
- Removal of brackets
- Rearrangement of terms of a conditionally convergent series
- Pringsheim’s Method
- Multiplication of infinite series
- Mertens’s theorem
- Abel’s theorem
- Failure of multiplication rule
- Tests for absolute convergence
- Abel’s inequality
- Dirichlet’s test
- Abel’s test
- Eilers constant
- The integral test for series of positive terms
- Convergence of Infinite Products
- Convergence and divergence of infinite products
- General principle of convergence of infinite products
- Weierstrass’s inequalities
- Absolutely Convergent Series
- Convergence of infinite products
- Rearrangement of factors
- Semi-convergent infinite products
- Complex factors
- Uniform Convergence of Sequences and Series of Functions
- Uniform Convergence
- Cauchy’s general principle of uniform convergence
- Dini’s criterion of uniform convergence of a sequence of continuous functions
- Tests for uniform convergence
- M-test
- Weierstrass’s M-test
- Abel’s test
- Dirichlet’s test
- Uniform convergence and continuity
- Uniform convergence and integration
- Uniform Convergence and Differentiation
- Uniform convergence of infinite products
- The...
...Contd: Infinite Series & Products
Weierstrass's Approximation Theorem • Arzelà's Theorem on Equicontinuous Families • Power Series • Definition and some elementary theorems • Radius of convergence • Uniform convergence of power series • Properties of power series • Abel's summability • Abel’s theorem • Tauber’s theorem • Expansions of Trigonometrical Functions as Infinite Series and Products • Infinite Product for sin x and cos x • Convergence of Infinite Products for sin x and cos x • Weierstrass’s Formula for sine as an infinite product • Series for sec x.

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Integral Transforms (Transform Calculus)
~A.R. Vasishtha & R.K. Gupta

- The Laplace Transform • Integral Transform (Definition) • Laplace Transform (Definition) • Linearity Property of Laplace Transform • Piece-wise (or sectionally) continuous functions • Existence of Laplace Transform • Functions of Exponential order • A function of Class A • Table (Laplace Transforms of some elementary functions) • First translation or shifting theorem • Second translation or shifting theorem • Change of scale property • Laplace transform of the derivative of F(t) • Laplace transform of nth order derivative of F(t) • Initial value theorem • Final value theorem • Laplace transform of Integrals • Multiplication by t • Multiplication by tⁿ • Division by t • Evaluation of Integrals • Periodic Functions • Some Special functions • Table Laplace Transform Theorems • The Inverse Laplace Transform • Null Function (Definition) • Lerch's Theorem • Linearity Property • Table of Inverse Laplace transforms • First translation or shifting theorem • Second translation or shifting theorem • Change of scale property • Use of Partial Fractions • Inverse Laplace transform of derivatives • Inverse Laplace transform of Integrals • Multiplication by powers of P • Division by powers of P • Convolution (Definition) • Convolution theorem • Heaviside’s expansion formula or formula • The Beta function • The Complex Inversion formula • Table of Inverse Laplace Transform theorems • Application of Laplace Transform to Solutions of Differential Equations • Solution of ordinary Differential Equations with constant coefficients • Solution of ordinary Differential Equations with variable coefficients • Solution of Simultaneous Ordinary Differential Equations • Solution of partial Differential Equations • Applications to Electrical circuits • Applications to Mechanics • Application of Laplace Transform to Integral Equations • Definitions • Applications of L.T. to Integral Equations • Applications of Laplace Transforms in Initial and Boundary Value Problems • A Boundary Value Problem • Heat Conduction Equation • Wave Equation • Laplace Equation • Applications to Beams • Miscellaneous Exercises • Fourier Transforms • Dirichlet’s Conditions • Fourier Series • Fourier Integral formula • Fourier Transform or Complex Fourier Transform • Inversion Theorem for Complex Fourier transform • Fourier sine transform • Inversion formula for Fourier sine transform • Fourier cosine transform • Inversion formula for Fourier cosine transform • Linearity property of Fourier transform • Change of Scale property • Shifting Property • Modulation Theorem • Multiple Fourier Transforms • Convolution • The Convolution or Falting theorem for Fourier transforms • Parseval’s Identity for Fourier Transforms • Relationship between Fourier and Laplace Transforms • Fourier transforms of the derivatives of a function • Problems related to integral equations • Finite Fourier Transforms • Finite Fourier sine transforms • Inversion formula for sine transforms • Finite Fourier cosine transform • Inversion formula for cosine transform • Multiple finite Fourier transforms • Operational properties of finite Fourier sine transforms • Theorem I • Theorem II • Operational properties of finite Fourier cosine transforms • Theorem III • Theorem IV • Combined properties of finite Fourier sine and cosine transforms • Convolution • Applications of Fourier Transforms in Initial and Boundary Value Problems • Application of infinite Fourier transforms • Choice of infinite sine or cosine transforms • Application of finite Fourier transforms • Finite Fourier transforms of partial derivatives • Choice of finite sine or cosine transforms • Hankel Transforms • Hankel Transform (Def.) • Inversion formula for the Hankel transform • Some Important Results for Bessel Functions • Linearity Property • Hankel Transform of the Derivatives of a function • Hankel transform of \[ \frac{d^2 f}{dx^2} + \frac{1}{dx} \frac{df}{dx} - \frac{n^2}{x^2} f \] • Parseval’s Theorem • The Finite Hankel Transforms • Finite Hankel Transform (Def.) • Another form of Hankel Transform • Hankel Transform of \[ \frac{df}{dx} \]

Hankel Transform of \[ \frac{d^2 f}{dx^2} + \frac{1}{dx} \frac{df}{dx} \]
where p is the root of the equation \[ J_p (ap) = 0 \] • Hankel Transform of \[ \frac{d^2 f}{dx^2} + \frac{1}{dx} \frac{df}{dx} - \frac{n^2}{x^2} f \] (x) where p is the root of the equation \[ J_p (ap) = 0 \] • Applications of Hankel Transform in Initial and Boundary Value Problems • Mellin Transform (Def.) • The Mellin inversion theorem • Linearity property • Some elementary properties of Mellin transform • Mellin transform of derivatives • Mellin transform of integrals • Convolution (or Falting) theorem for Mellin transform.

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Linear Algebra (Finite Dimensional Vector Spaces)
~J.N. Sharma, A.R. Vasishtha & A.K. Vasishtha

- Vector Spaces • Binary operation on a set • Group Definition • Field • Vector space • General properties of vector spaces • Vector subspaces • Algebra of subspaces • Linear combination of vectors, Linear span of a set • Linear sum of two subspaces • Linear combination of vectors, Linear span of a set • Linear sum of two subspaces • Linear dependence and linear independence of vectors • Basis of a vector space • Finite dimensional vector spaces • Dimension of a finitely generated vector spaces • Dimension of subspace • Homomorphism of vector spaces of Linear Transformation •

Contd...
Linear Difference Equations

Introduction

Applications of Difference Equations
Study of Period of Analysis
Verbal Learning Experiment
Panel Surveys
Social Sciences
Psychology
Physiology
Economic Dynamics
The Calculus of Finite Differences
Definitions
Operators
Linear Operators
Algebra of Operators
First Differences (or Forward Differences) of y
Forward Difference Operator Δ
Backward Difference Operator \( \Delta^{-1} \)
Central Difference Operator \( \delta \)
Second and Higher Order Differences
Identity Operator
Second and Higher Order backward Differences
The Transition or Shifting Operator E
Properties of Δ and E
Equivalence of Operators
Some Important Theorems of Δ and E
To Express any Functions In terms of Leading Term and the Leading Differences of Difference Table
Leibnitz’s Rule for Differences
Factorial Function
The Difference of Functional Factor
Method of Representing any Polynomial in Factorial Notation
Indefinite Summation, The operator \( \Delta^{-1} \)

Theorem. To prove \( \Delta^{-1}y = Y + t(x) \) i.e To find \( \Delta^{-1} x^{1/n} \) i.e To prove \( \Delta^{-1} c^x = \frac{c^x}{c^x - 1} + t(x) \) i.e To prove \( \Delta^{-1}[V(y).ΔU(x)] =U(x).V(x) - Δ^{-1}[EU(x)] \)

\( \Delta V(x) \) i.e Analogies between the Difference and Differential Calculus
Difference Equations
To Write a Difference Equation as a Relation among the Value of y
Linear Difference Equation
Order of a Linear Difference Equation
Solution of a Difference Equation
An Existence and Uniqueness Theorem
Solution of the Equation \( y_{n+1} = Ay_n + C \)
Theorem
Sequences
Definitions
Solution as Sequences
Theorem A Probability Model for Learning
Approximating a Differential Equation by a Difference Equation
Linear Difference Equations with Constant Coefficients
Basic Definitions
Theorem 1
Finite Linear Combination of Solutions
Theorem 2
Theorem 3
Fundamental Set of Solutions (or Linearly Independent Solns.)
Theorem
General Solution of the Hom. Diff. Equation of Order 2
General Solution of the Hom. Diff. Equation of Order n
Particular Solution of the Complete Diff. Equation
Method of Undertermined Coefficients to Find the Particular Solution
Special operator Method to Find the Particular Solution
Method of Variation of Parameters
Solution of Simultaneous Difference Equations
Matrix Method for Solving a system of linear diff. equations
Working Method for Solving a Second Order Homo. Difference Equation with Constants Coefficients by Matrix Method
Examples from the Social Sciences
The First Order Equations. Cobweb Cycles and Generating Functions
Solution of \( y_{n+1} - b_ny_n = f_n \)
Cobweb Phenomenon (or Cobweb Cycles)
Generating Functions
Some Special Generating Functions
The Linearity Property of the Generating Function Transformation
Generating Function Method for Solving a Linear Difference Equation.
...Contd: Integral Equations

then for a Proper Choice of $\phi_0 \phi(x) = D(x, \xi, \lambda_0)$ is a Continuous Solution of the Homogeneous Integral Equation.

Fundamental Functions, Integral Equations with Degenerate Kernels, Hilbert Schmidt Theory, All Iterated Kernels of a Symmetric Kernel are also Symmetric, Orthogonality of Fundamental Functions, Eigen Values of Symmetric Kernel are Real, Real Characteristic Constants, Expansion of a Symmetric Kernel in Eigen Functions, Symmetric Kernels with a Finite Number of Eigen Values, Symmetric Kernels with a Finite Eigen Values $\lambda_{m+1} \lambda_{m+2}$, etc.


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Linear Programming

- R.K. Gupta

- Mathematical Preliminaries, Matrices and Determinants, Operations of Matrix and Addition and Multiplication, Sub- matrix, Minor of order $k$, Determinant, Important properties of determinants, Minors, Cofactors, Rank of a Matrix, Adjoint of a Matrix, Singular and Non-singular Matrices, Inverse of a matrix, Vectors and Vector Spaces, Definitions, Euclidean space, Linear Dependence and Independence of vectors, Linear Combination (L.C.) of vectors, Spanning Set, Basis Set, Some useful Theorems of Linear Algebra, Simultaneous Linear Equations, Linear Programming Problems, Formulation and Graphical Solution, General Linear Programming problems, Mathematical formulation of a L.P.P., Basic Solution (B.S.), An Important Theorem, Some Important Theorems, Solution of a linear programming problem, Geometrical (or graphical) method for the solution of a L.P.P., Convex Sets and their Properties, Definitions, Convex Combination, Some Important Theorems, Simplex Method, Slack and Surplus Variables, Some Definitions and Notations, Fundamental Theorem of Linear Programming, To obtain B.F.S. from F.S., To Determine Improved B.F.S., Unbounded Solutions, Optimality Conditions, Alternative Optimal Solutions, Inconsistency and Redundancy in Constraint Equations, To determine starting B.F.S., Computational procedure of the simplex method for solution of a maximization L.P.P., Artificial Variables Technique, L.P.P. with unrestricted variables, Sol. of system of simultaneous linear eqs. by simplex method, To compute the inverse of a matrix for which one column is different from that of a matrix whose inverse is known, Inverse of a matrix by Simplex Method, Resolution of Degeneracy, Conditions for the occurrence of degeneracy in a L.P.P., Method of Resolving Degeneracy, Charnes’ Perturbation Method, Selection of the outgoing (departing) vector, Computational Procedure, Generalized Simplex Method, Revised Simplex Method, Revised Simplex Method in standard Form I (Formulation of a L.P.P. in the form of revised simplex), Notations for Standard form I, To find the inverse of the Basis and the Basic solution in standard Form I, Computational Procedure of the revised Simplex Method in Standard Form I, Revised Simplex Method in Standard Form II, Notations, Basis and its Inverse in Standard form II, Computational Procedure of the Revised Simplex Method in Standard Form II, Advantages and Disadvantages of Revised Simplex Method over the original Simplex Method, Duality, Symmetric Dual Problem, Unsymmetric Dual Problem, The dual of a mixed system, Standard form of the primal, Theorem Dual of the dual of a given primal is the primal itself, Fundamental Properties of Dual Problems, Complementary Slackness Theorem, Correspondence between primal and dual, To read the solution of the dual from the final Simplex table of the primal and vice versa, Dual Simplex Algorithm, Derivation of the Dual Simplex Algorithm, Initial solution for Dual Simplex Algorithm, Advantage of Dual Simplex Algorithm, Computational Procedure of the Dual Simplex Algorithm, Primal Dual Algorithm, To determine the

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initial Dual Solution ● To determine the Restricted Primal Problem ● To find the Entering and leaving vectors ● Method to obtain New Dual Solution ● Test of optimality ● Computational Procedure of Primal-Dual Algorithm ● Sensitivity Analysis ● Variation of a price vector c ● Variation in the requirement vector b ● Variation in the component $a_{ij}$ of the coefficient matrix A ● Addition of a new variable to the problem ● Addition of a new constraint to the problem ● Parametric Linear Programming ● Linear Variation in c ● Linear Variation in b ● Integer Programming ● Importance (or need) of I.P.P. ● Solution of I.P.P. ● Gomory’s all I.P.P. method ● Construction of Gomory’s constraint ● Computation procedure for the solution of all I.P.P. by Gomory method ● The Branch and Bound Technique ● Branch and Bound Algorithm ● Assignment Problem ● Important theorems ● Method for solving an assignment Problem (Assignment algorithm) ● Unbalanced Assignment Problem ● Transportation Problem ● Difference between a transportation and an Assignment problem ● Few Important definitions ● Solution of transportation problem ● To find an Initial feasible solution ● Optimality Test ● Theorem ● Computational Procedure of optimality test ● Transportation Algorithm or Modi Method ● Degeneracy in Transportation Problems ● Unbalanced Transportation Problem ● Game Theory (Competitive Strategies) ● Competitive Games ● Finite or Infinite Games ● Zero sum game ● Two person zero sum (or Rectangular) Games ● Pay-off matrix ● Strategy ● Solution of a Game ● Minimax and minimax criterion of optimality ● Solution of a rectangular game with saddle point ● Solution of a rectangular game in terms of mixed strategies ● Important properties of optimal mixed strategies ● Solution of $2 \times 2$ games without saddle point ● Dominance property ● Graphical method for solution of $2 \times n$ and $2 \times m$ games ● Algebraic method for Approximate Solution ● Equivalence of the rectangular matrix game and linear Programming ● Fundamental theorem of Game theory (Minimax Theorem) ● Solution of a rectangular game by simplex method ● Summary of methods for solving the rectangular games ● Minimax and Maximin of a function of several variables ● Saddle points of a function of several variables ● Necessary and sufficient condition for a function $E(x, y)$ to possess a saddle point.
...Contd: Mathematical Analysis-II

Morgan and Bertrand's Test
- An alternative to Bertrand's Test
- Summary of Tests
- General Series
- General Principle of Convergence
- Alternating series
- Absolute and non-absolute Convergence
- Re-arrangement of Terms of an Absolutely Convergent Series
- Inversion of parentheses
- Removal of Brackets
- Re-arrangement of terms of a Conditionality Convergent Series
- Pringsheim’s Method
- Multiplication of Infinite Series
- Merten’s Theorem
- Abel’s Theorem
- Failure of Multiplication Rule
- Test for Absolute Convergence
- Abel’s Inequality
- Dirichlet’s Test
- Abel's Test
- Euler’s Constant
- Integral Test for Series of Positive Terms
- Open and Closed Sets of Real Numbers
- Neighbourhoods
- Open Sets
- The Structure of Open Sets in R
- Closed Sets
- Accumulation Points, Adherent Points
- Closed Sets and Accumulation Points
- Closure
- Interior, Exterior and Boundary of a Set
- Dense, non-dense, perfect and isolated Sets
- Covering Theorems
- Compactness
- Structure of Closed Sets on the real line
- Cantor’s Ternary Set
- Limits and Continuity
- Definitions
- Limits
- Algebra of Limits
- Continuity
- The four functional limits at a point
- Kinds of discontinuities
- Saltus
- Theorems on continuity
- Theorems on discontinuous functions
- Pointwise discontinuous functions
- Uniform continuity
- Absolute Continuity
- Continuity of the inverse function
- Some more examples on continuity
- Differentiability
- Derivative at a point
- Progressive and regressive derivatives
- Differentiability in [a,b]
- Derivative of a function
- Meaning of sign of derivative
- Geometrical meaning of a derivative
- A necessary condition for the existence of a finite derivative
- Algebra of derivatives
- The chain rule
- Derivative of an inverse function
- Darboux Property
- Rolle’s Theorem
- Lagrange’s Mean value theorem
- Deductions from mean value theorem
- Cauchy’s mean value theorem
- Taylor's development of a function in a finite form with Lagrange’s form of reminder
- Taylor’s theorem with Cauchy’s form of reminder

The Riemann Integral
- Sets of measure zero
- Partitions and Riemann Sums
- Upper and Lower R-integrals
- R-integrability
- Riemann’s necessary and sufficient conditions for R-integrability
- Some classes of integrable Calculu
- Mean Value Theorems
- Integration by Substitution
- Integration by Parts
- The integral as a limit
- The Riemann Stieltjes Integral
- A generalization of the Riemann Integral
- Partitions
- Lower and upper Riemann-Stieltjes sums
- The lower and upper Riemann-Stieltjes Integrals
- The Riemann Stieltjes Integral
- The QS-integrals as a limit of sums
- Some classes of RS-integrable functions
- A relation between R-integral and RS-integral
- Integration of vector valued function
- Some more theorems on Integration
- Convergence of Improper Integrals
- Improper Integral
- Integral with infinite limits
- Test for the convergence of \( \int_a^b f(x) \, dx \)
- Comparison Test
- To test the convergence of \( \int_a^b \frac{dx}{x^2} \)
- The \( \mu \)-test
- Abel's test
- Dirichlet's test
- Absolute convergence
- Test for the convergence of improper integral \( \int_a^b f(x) \, dx \)
- Comparison test
- To test the convergence of \( \int_a^b \frac{dx}{(x-a)^n} \)
- The \( \mu \)-test
- Abel's test

Dirichlet's test
- Operation with improper integrals
- Metric Spaces
- Euclidean spaces
- Metric spaces
- Neighbourhoods
- Limit points
- Open and closed sets
- Connectedness
- Compactness
- Completeness
- Cantor's Intersection Theorem
- Baire category Theorem
- Completeness and Contracting Mapping
- Limits and Continuity
- Functions of Several Variables
- Continuity of Functions of two Variables
- Partial Derivatives
- Interchange of the Order of Differentiation
- Differentiability of two variables
- Composite Functions
- Linear transformations
- Matrices
- Differentiation
- Partial Differentiation
- The Inverse Function Theorem
- The Implicit Function Theorem
- Jacobians
- Definition
- Case of Functions of Functions
- Jacobian of Implicit Functions
- Necessary and sufficient condition for a Jacobian to vanish
- Convolutions and Invariants

Beta and Gamma Functions
- Principal and general value of an improper integral
- Infinite limits
- To find the value of \( \int_0^1 \frac{dx}{x^a} \)
- Test for the convergence of \( \int_0^1 \frac{dx}{x^a} \)
- To find the value of the integral \( \int_0^1 \frac{x^m}{1+x^n} \, dx \)
- To find the value of \( \int_0^1 \frac{x^n}{1-x^n} \, dx \)
- Deductions from \( \int_0^1 \frac{x^m}{1+x^n} \, dx \)
- \( \int_0^1 \frac{x^n}{1-x^n} \, dx \)
- Method of differentiation under the integration sign
- Method of integration under the integration sign
- Euler’s Integrals—Beta and Gamma Functions
- Elementary properties of Gamma Functions
- Transformations of Gamma Functions
- Another form of Beta Function
- Relation between Beta and Gamma functions
- Other transformations
- To prove that \( \Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{m-\frac{1}{2}}} \Gamma(2m) \)
- \( \int_0^1 \frac{dx}{(x-a)^n} \)

Double and Triple Integrals, Dirichlet’s Theorem
- Double Integrals
- Second order element in polar curves
- Multiple Integrals
- Area of the surface
- Dirichlet’s Theorem
- Liouville’s Extension of Dirichlet’s Theorem
- Change of order of integration
- Transformation of multiple Integrals
- Transformation for implicit functions
- Transformation of element of surface
- Volumes and Surfaces
- Polar Coordinates
- Examples on Surfaces
- Uniform Convergence of Sequences and Series of Functions
- Uniform Convergence
- Cauchy’s general principle of uniform convergence
- Dini’s Criterion for uniform convergence of a sequence of continuous functions
- Tests for uniform convergence
- Uniform convergence and continuity
- Uniform convergence and integration
- Uniform convergence and differentiation
- Everywhere continuous but nowhere differentiable functions
- Weierstrass’s non-differentiable function
- The Weierstrass’s Approximation Theorem
- Stone-Weierstrass Theorem
- Arzela’s Theorem on Equicontinuous Families
- Power Series
- Definition
- Cauchy’s theorems on limits
- Radius of convergence
- Uniform convergence of power series
- Properties of Power Series
- Abel’s Summability
- Differentiation and Integration of Vectors
- Vector function
- Limit and continuity of a vector function
- Derivative of a vector function with respect to a scalar
- Curves in space
- Velocity and acceleration
- Integration of vector functions
- Gradient, Divergence and Curl
- Partial derivatives of vectors
- The vector differential operator Del, \( \nabla \)
- Gradient of a scalar field
- Level Surfaces
- Directional derivative of a scalar point function
- Tangent plane and normal to a level surface
- Divergence of a vector point function
- Curl of a vector point function
- The Laplacian operator \( \nabla^2 \)
- Important vector identities
- Invariance
- Green’s, Gauss’s and Stoke’s Theorems
- Some preliminary concepts
- Line integrals
- Circulation
- Surface integrals
- Volume integrals
- Green’s theorem in the plane
- The divergence theorem of Gauss
- Green’s theorem
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- Line integrals independent of path
- Physical interpretation of div. and curl.
Contents

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(Measure Theory & Functional Analysis)

- Basic Concepts of Set and Basic Operations
- Concepts of Set
- Notation
- Set of Sets
- Subset
- Super Set
- Equality of Sets
- Proper Subset
- Finite Set
- Infinite Set
- Null Set
- Power Set
- Universal Set
- Indexed Set and Index Set
- Hereditary Property
- Pairwise Disjoint
- Set Operations
- Union
- Intersection
- Disjoint Sets
- Difference of Sets
- Complement of a Set
- Symmetric Difference of Sets
- Distributive Law
- De-Morgan’s Law
- Ordered Pair
- Equality of Ordered Pairs
- Product of Sets
- Product Sets in General
- Functions and Sequences
- Function
- Onto and Into Mappings
- One-one and Many-one Mappings
- Real Valued Map
- Set Function
- Real Valued Set Function
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- Sequence
- Convergent Sequence
- Bounded Sequence
- Metric Space
- Monotonic (Increasing & Decreasing)
- Axiom of Choice
- Choice Function
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- Zermelo’s Postulate
- Chain
- Finite Character
- Hausdorff Maximal Principle
- Tukey’s Lemma
- Zorn’s Lemma
- Well-ordering Theorem
- Kuratowski Lemma
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- The Totally Ordered Set
- Subset of a Ordered Set
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- Principle of Transfinite Induction
- Ordinal Number
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- Bounded Sets, Derived Sets,
- Open Sets and Closed Sets on the Real Line
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- Continuity
- Bounded Linear Set
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- Measure of a Paralleloiped
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- Lebesgue Measureable Set
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- Limiting Sets
- Covering in the Sense of Vitali
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- The Lebesgue Integral of a Function
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- Indefinite Integral and Differentiation
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- Dedekind's theory of real numbers
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- Cantor’s Theory of Real Numbers
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- Definition
- General properties of...
The primary focus of the page is on matrices, including topics such as the definition of matrices, properties of matrices, and theorems related to matrices. The page also touches on other mathematical concepts such as number theory, linear dependence and independence of vectors, characteristic roots, and orthogonality.

Here is a structured summary of the content:

### Matrices

- **Basic Concepts:** Matrix, Submatrix, Row, Column, Matrix Rank, Determinant, Minor, Cofactor, Adjoint Matrix
- **Elementary Operations:** Addition, Multiplication, Cyclic and Complementary Subspace
- **Field Extensions:** Algebraic and Simple Field Extensions, Normal Extension
- **Euler's Theorem:** Euler's theorem, Eigenvectors and Eigenvalues, Normal Matrices, Hermitian Matrices
- **Diagonalization:** Diagonalizable Matrices, Orthogonally Similar Matrices, Quadratic Forms
- **Annihilators:** Annihilators of Matrices
- **Galois Theory:** Field Extensions, Finite Field Extensions
- **Matrix Polynomials:** Matrix Polynomials, Companion Matrix
- **Eigenvalues and Eigenvectors:** Characteristic Values, Characteristic Vectors, Cayley-Hamilton Theorem
- **Orthogonal Matrices:** Orthogonal Matrices, Orthogonality, Unitary Matrices
- **Orthogonal Vectors:** Orthogonal Vectors, Inner Product of Vectors
- **Linear Transformations:** Linear Transformations, Operators, Isomorphisms of Linear Spaces
- **Submodules:** Submodules of Modules, Homomorphisms

### Mathematical Methods

- **Spherical Harmonics:** Spherical Harmonics, Kelvin's Theorem
- **Legendre's Equation:** Legendre's Equation, Legendre Polynomials
- **Bessel's Equation:** Bessel's Equation, Bessel Functions
- **Laplace's Equation:** Laplace's Equation, Laplace's Integral
- **Bessel's Functions:** Bessel Functions, Orthogonal Properties of Bessel Functions
- **Recurrence Relations:** Recurrence Relations, Beltrami's Results
- **Series Expansions:** Series Expansions, Rodrigues' Formula
- **Special Functions:** Special Functions, Generalized Functions
- **Number Theory:** Number Theory, Divisibility, Congruence
- **Linear Algebra:** Linear Algebra, Vector Spaces, Linear Transformations

### Additional Notes

- **The Fundamental Theorem of Arithmetic:** Theorem, Proof
- **Wilson's Theorem:** Wilson's Theorem, Proof
- **Euclid's Lemma:** Euclid's Lemma, Proof
- **Least Common Multiple:** LCM, Properties
- **Greatest Common Divisor:** GCD, Properties

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The page emphasizes the importance of understanding the underlying concepts and theorems related to matrices and their applications in various fields of mathematics.
• If \( v \) is a solution of Legendre’s Equation then \( (1 - x^2)^{n/2} \frac{d^n y}{dx^n} \) is a solution of associated Legendre’s Equation.
• Associated Legendre’s Function.

Properties of the associated Legendre’s Function.
• Orthogonal properties of associated Legendre’s Functions.
• Recurrence formulae for associated Legendre’s Functions.
• Trigonometrical Series for \( P_n(x) \).

Legendre’s Functions of the Second Kind \( Q_n(x) \).
• Legendre’s Functions of the second kind.
• Neumann’s Integral.
• Recurrence formulae for \( Q_n(x) \).
• Relation between \( P_n(x) \) and \( Q_n(x) \).
• Christoffel’s Second Summation formula.
• Assuming \( P_n(x) \) as a solution of Legendre’s Equation, show that the complete solution of this Legendre’s Equation is \( AP_n(x) + BQ_n(x) \).

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• The Hypergeometric Series.
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• Integral formula for the Hypergeometric Function.
• Kummer’s Theorem.
• Gauss’s Theorem.
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• Differentiation of Hypergeometric Function.
• The confluent Hypergeometric Functions.
• theorem.
• Whittaker’s confluent Hypergeometric Function.
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• Integration of Bessel’s Equation in series for \( n = 0 \).
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• Recurrence Formulae for \( J_n(x) \).
• Generating function for \( J_n(x) \).
• A second solution of Bessel’s Equation.
• Hermite Polynomials.
• Hermite Differential Equation.
• Solution of Hermite Equation.
• Hermite’s Polynomials.
• Generating Function.
• Other forms for the Hermite Polynomials.
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• Recurrence formulae for Laguerre Polynomials.
• Associated Laguerre’s Equation.
• If \( v \) is a solution of Laguerre’s Equation of order \( n + \alpha \), then \( \frac{d^n y}{dx^n} \) satisfies Laguerre’s Associated Equation.
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• Associated Laguerre’s Polynomials \( L_n^\alpha(x) \).
• Generating function.
• Other form for associated Laguerre’s Polynomial.
• Orthogonal properties of the associated Laguerre Polynomials.
• Recurrence formulae for the associated Laguerre Polynomials.

Chebyshev Polynomials.
• Chebyshev’s Diff. Equation.
• Chebyshev Polynomials.
• To prove that \( T_n(x) \) and \( U_n(x) \) are independent solutions of Chebyshev’s Equation.
• Relations for \( T_n(x) \) and \( U_n(x) \).
• Generating Function.
• Orthogonal properties of Chebyshev Polynomials.
• Recurrence formulae for \( T_n(x) \) and \( U_n(x) \).

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• Definitions.

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• Other Type of orthogonality.
• Strum-Liouville Equation.
• Theorem.
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• Laplace’s Definite integrals for \( P_n(x) \).
• Orthogonal properties of Legendre’s Polynomials.
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• Even and odd functions.
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• An important case.
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• If \( v \) is a solution of Legendre’s equation then \( (1 - x^2)^{m/2} \frac{d^m v}{dx^m} \) is a solution of associated Legendre’s equation.
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Properties of the associated Legendre’s Function.
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Contd...
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...Contd: Vector Algebra
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Inequalities • Probability Function of a Quotient • Change of variable • Bivariate Distributions • Marginal Distributions • Conditional Probability Density • Stochastic Independence • Important Theoretical Distributions • Theoretical Distributions • Binomial distribution • Pascal’s Triangle • Moments of the Binomial Distribution • Proof of the formula Mk = \( p^n \frac{d^n}{dp^n} (p + q)^n \) • Mode of Binomial Distribution • Recursion formula for binomial distribution \( \mu_{r+1} = pq \left( n\mu_{r-1} + \frac{d\mu_r}{dp} \right) \) • Poisson Distribution • Poisson Process • Mode of the Poisson Distribution • Constants of the Poisson Distribution • Multinomial Distribution • Hyper-geometric Distribution • Mean and variance of the Hyper-geometric Distribution • Normal Distribution • Derivation of Normal Distribution • Properties of Normal Distribution • Constants of the Normal Distribution • Some further properties of the Normal Distribution • Probable Error • Importance of the Normal Distribution • Fitting a Normal Distribution • Central Limit Theorem • The Law of Large Numbers • Weak law of Large Numbers • Moment Generating Functions and Cumulants • Expectation of a Random Variable • Expectation of Functions of Random Variable • Expectation of Functions of two Random Variables • Moment Generating Function • Change of origin and scale in Moment Generating Function • M.G.F. of a sum • Binomial Distribution • Poisson Distribution • Negative Binomial Distribution • Normal Distribution • Sum of Independent Normal variates • Cumulants • Additive Property of Cumulants • Factorial Moments • Sum of Poisson variates • Characteristic Function • Inversion • Cauchy’s Distribution • Probability Generating Function • Properties of Characteristic Functions • Some Characteristic Functions • Probability Generating Functions • Relation between Probability Generating Function and Characteristic Function • Factorial Moment Generating Functions • Method of Least Squares and Curve Fitting • Method of Least squares • Some Special Curves • Bivariate Distribution, Regression and Correlation • Scatter or Dot Diagram • \( r \) independent of Origin and Scale • Sterograms and Collection Surface • Probable error of coefficient of Correlation • Rank Correlation • \( r \) lies between –1 and 1 • Variance of a sum or difference • Regression • Range of \( r \) • Correlation ratio • Linear Relationship • Causation and effect • Regression and Correlation • Multiple and Partial Correlation • Multiple Correlation and Partial Correlation • Equation of the Regression Plane • Multiple Correlation Coefficient • Partial Correlation Coefficient • Consistence of Data and Association of Attributes • Attributes • Classification with reference to attributes • Class frequencies • Relation between class frequencies • Consistence of data • Independence and Association of attributes • Yule’s coefficient of Association • Finite Differences and Interpolation • Difference Table • Some Nomenclatures • E and \( \Delta \) notation • Factorial Notation • Interpolation • Algebraic Methods of Interpolation • Newton’s Formula for equal Intervals • Lagrange’s Formula • Central Differences • Gauss’s Backward Formula • Gauss’s Forward Formula • Bessel’s Formula • Stirling’s Formula • Distinction between Interpolation and Extrapolation • Divided Differences Formula • Newton’s Divided Differences Formula • Preliminary Concepts on Sampling • Universe, definition • Sampling: Types of samples • Simple Sampling • Devices for Random Sampling • Tippett’s Numbers • Stratified Sampling • Simple Sampling of Attributes-Large Samples • Population and Samples • Simple
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  - Special Continuous Probability Distributions
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  - Simultaneous Linear Equation
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  - Conditions for a Maximum or Minimum of f(x)
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- Generating Functions
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  - Infinite Production Rate and having no Shortages
  - Model III: Economic Lot-size Model with Uniform Rate of Demand, Infinite Rate of Replenishment having no Shortages
- Deterministic Models With Shortages
  - Model IV: Fixed Time Model
  - Model V: Economic Lot-size Model with Uniform Rate of Demand, Infinite Rate of Demand
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...Contd: Operations Research

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Method  • Construction of Gomory’s Constraint and Gomory’s Cutting Plane  • All-Integer Cutting Plane Algorithm  • Mixed-Integer Cutting Plane Algorithm  • The Branch and Bound Technique  • Branch and Bound Algorithm  • Assignment Problem  • Important Theorem  • Hungarian Method (Reduced Matrix Method)  • Unbalanced Assignment Problems  • Maximization Assignment Problem  • Restrictions on Assignment  • Transportation Problem  • Difference between a Transportation and an Assignment Problem  • Solution of a Transportation Problem  • To Find an Initial Feasible Solution  • Optimality Test  • Theorem  • Computational Procedure of Optimality Test  • Transportation Algorithm or MODI (Modified Distribution) Method  • Degeneracy in Transportation Problem  • Unbalanced Transportation Problems  • Profit Maximization Problems  • Prohibited Transportation Route  • Sequencing (Including Travelling Salesman Problem)  • A Sequencing Problem  • General Assumptions  • Sequencing Decision Problem for n-Jobs on two Machines  • Sequencing Decision Problem for n-Jobs on Three Machines  • Sequencing Decision Problem for n-Jobs on m Machines  • Processing Two Jobs Through m Machines  • Graphical Method  • Travelling Salesman (or Routing) Problem  • Dynamic Programming  • Bellman’s Principle of Optimality in Dynamic Programming  • Multistage Decision Problem  • Characteristics of Dynamic Programming Problems  • Solution of a Multi-stage Problem by Dynamic Programming with Finite Number of Stages  • Solution of Linear Programming Problem as a Dynamic Programming Problem  • Solution of an Inventory Problem as a Dynamic Programming Problem  • Game Theory (Competitive Strategies)  • Competitive Games  • Finite and Infinite Games  • Zero Sum Game  • Two Person Zero Sum (or Rectangular) Games  • Pay-off Matrix  • Strategy  • Solution of a Game  • Maximin and Minimax Criterion of Optimality  • Saddle Point  • Solution of a Rectangular Game with Saddle Point Y Solution of a Rectangular Game in terms of Mixed Strategies Y Important Properties of Optimal Mixed Strategies  • Solution of 2 × 2 Games Without Saddle Point  • Dominance Property  • Arithmetic Method (or the Method of Oddments or the Short Cut Method) for the Solution of 2 × 2 Game without Saddle Point  • Graphical Method for the Solution of (2 × n) and (m × 2) Games  • Algebraic Method for the Solution of a General Game  • Iterative Method for Approximate Solution  • Equivalence of the Rectangular Matrix Game and Linear Programming  • Fundamental Theorem of Game Theory (Minimax Theorem)  • Solution of a Rectangular Game by Simplex Method  • Matrix Method for \( n \times n \) (i.e., Square) Games  • Summary of Methods for Solving the Rectangular (Two Person Zero Sum) Games  • Minimax and Maximin of a Function of Several Variables  • Saddle Point of a Function of Several Variables  • Necessary and Sufficient
...Contd: Operations Research


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Rigid Dynamics-I (Dynamics of Rigid Bodies)

—P.P. Gupta & G.S. Malik

● Moment of Inertia ● Definitions ● Moments of inertia in some simple cases ● Theorem of parallel axes ● Pappus theorem (solid generated); to find the M.I. of the body ● Pappus theorem (surface generated); to find the M.I. of the body ● Moment of inertia about a line ● To find M.I. about any axis which passes through the intersection of two perpendicular axes in the plane provided M.I.’s and P.I.’s about these axes are known ● Elementary theorem on moment of inertia ● Method of differentiation ● M.I. of heterogeneous bodies ● Momental ellipsoid ● Momental ellipse ● Bodies ● Equipmomental Theories ● Principal axes ● Principal moments ● D’Alembert’s Principle ● Motion of a particle and a Rigid body ● Impressed and Effective forces ● D’Alembert’s Principle ● Angular Momentum ● General equations of Motion ● Linear momentum ● Motion of the Centre of Inertia ● Motion about Centre of inertia ● Impulsive forces ● Impulsive forces continued ● Application of D’Alembert’s Principle to impulsive forces and general equation of motion ● Motion about a Fixed Axis ● Moment of the effective forces about the axis of rotation ● Kinetic Energy ● Equation of Motion ● Compound Pendulum ● Centre of Suspension ● Centre of Suspension and oscillations are Convertible ● Minimum time of oscillation of compound pendulum ● Reaction of the axis of rotation ● Motion about a fixed axis (impulsive forces) ● Centre of Percussion ● Centre of Percussion of a rod ● General case of centre of percussion ● Motion in Two Dimensions ● Equation of motion ● Kinetic energy ● Moment of Momentum ● Motion of a solid sphere down an inclined plane ● Slipping of rods ● A uniform straight rod sliding down in a vertical plane, its ends being in contact with two planes, one horizontal and other vertical ● Motion of a solid sphere down an inclined plane when rolling and sliding are combined ● Motion of a circular disc ● When two bodies are in contact, then to determine whether the relative motion involves sliding at the point of contact ● A sphere of radius “a” where C.G. is at a distance c from its centre C is place on a rough plane so that C.G. is horizontal; show that it will begin to roll or slide according as μ < or > \( \frac{ac}{k^2 + a^2} \) where k is the radius of a sphere about a horizontal axis through G. If μ equal to this value what happens? ● Motion of one sphere over another sphere which if fixed ● Motion of solid cylinder insider a hollow cylinder ● Motion of one body on another, when the lower body is free to turn about its axis ● Motion of one body on another when both bodies are free to move ● Motion in Two Dimensions (Under Impulsive Forces) ● To obtain the equations of motion of a rigid body under impulsive forces ● A rod of length 2a is held in a position inclined at an angle α to the vertical and is the left fall on the smooth inelastic horizontal plane will have it immediately after the impact if the height through which the rod falls is greater than \( \frac{1}{18} a \sec \alpha \cos \epsilon \cos \alpha (1 + 3 \sin^2 \alpha)^2 \) ● An imperfectly elastic sphere impinging on a fixed plane ● Work done by an Impulse ● Conservation of Momentum and Energy ● Principle of conservation of linear momentum (Finite Forces) ● Principle of Angular Momentum ● Conservation of Linear Momentum (Impulsive forces) Conservation of Angular Momentum (Impulsive forces) ● Principle of conservation of energy ● Vis-Viva ● Principle of Vis-Viva ● Conservative forces ● Theorem: When a body moves under the action of a system of conservative force, the sum of its kinetic and potential energies is constant throughout the motion ● The kinetic energy of a Rigid body, moving in any manner is at any instant equal to the kinetic energy of the whole mass, supposed to be collected at its centre of inertia and moving with it, together with the kinetic energy of the whole mass relative to its centre of inertia ● Initial Motion ● Definition ● Lagrange’s Equations of Motion, Small Oscillations, Normal Co-ordinates ● Generalised Coordinates ● Degrees of freedom ● Transformation of equations ● Classification of mechanical systems ● Kinetic energy and generalised velocities ● Generalised forces ● Lagrangian equations ● Lagrangian function ● Generalised momentum ● Kinetic energy as a quadratic function of velocities ● To reduce the principle of energy from the Lagrange’s equations (Conservative field) ● Small oscillations ● Lagrange’s equations with impulsive forces ● Euler Dynamical Equations ● Moving axes and the fixed axes ● Euler dynamical equations ● Kinetic energy of a Rigid Body about a fixed point ● Euler’s equations (impulsive forces) Eulerian angles and geometrical relations ● Instantaneous axis to rotation ● Invariable line ● Locus of the invariable line ● Deduction of Euler’s equations from Lagrange’s equations ● Hamiltonian Formulation and Variational Principles ● Hamilton’s form of the equations of motion ● Physical
significance of the Hamiltonian • Passage from the Hamiltonian to the Lagrangian • Variational methods • Techniques of Calculus of variations • Brachistochrome Problem • Extension of the variational methods • Hamilton’s variational principles • Derivation of Hamilton’s Equations from the variational principle • Principle of least action • Distinction in between Hamilton’s principle and principle of least action • Deduction of Hamilton’s principle using D’Alembert’s principle • Extension of Hamilton’s principle to non-conservative and non-holonomic systems • Motion of Top • Definition • Equation or motion of a top (Derived from Euler’s equations) • Equation of motion of a top (deduced from the principle of energy and momentum) • Equation of motion of top (deduce from Lagrange’s equations) • Steady motion • Stable motion (axis vertical) • Stable motion (axis is not vertical) • Limits of θ.

Rigid Dynamics-II (Analytical Dynamics) —P.P. Gupta & Sanjay Gupta

• Mechanics of a particle • Velocity of a particle • Acceleration of a particle • Linear momentum of the particle • Moment of force (or torque) and angular momentum of the particle • Work done by the force acting on a particle and kinetic energy • Power • Impulse • Conservative force and force field • Conservative theorem for a particle • The equation of motion of a particle D’Alembert’s Principle • Motion of a particle under resisting force • Motion in a resisting medium • The simple Harmonic oscillator • Damped Harmonic oscillator • Two and three dimensional harmonic oscillator • To discuss the motion of a particle executing harmonic vibrations and to find its orbit and frequency • Forced Harmonic oscillator • Central Force Field Motion • Central forces • Central orbit • $h = pu$ • To obtain the law of force, velocity and period time when the orbit is an ellipse • General features of the centre force field motion • Equations of motion for a particle in a central force field • Conservation of energy for central force field • Orbit under a central force (contd.) • Determination of the central force • Kepler’s Laws of planetary motion • bounded motion under in an inverse square field • First integrals of the two body motion under a central force • Reduction of two body problem to one body problem • Orbit under inverse square law • Stability of a nearly circular orbit • Unbounded motion-scattering in a central forces field • Rutherford’s scattering • Centre of mass and Laboratory co-ordinates • Transformation of scattering data from $C$-system to $L$-system • Motion of a System of Particles • D’Alembert’s Principle • Linear momentum of a system of particles • Torque on a system of particles • Angular momentum of a system of particles • Kinetic energy of a system of particles • Potential energy of a system of particles • Conservation of energy for the system of particles • Collision Problems • Lagrangian-Dynamics • Constraints and Generalised co-ordinates • 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rigid with a fixed point • Kinetic energy of body in general • Angular momentum of particle and of a system of particles • Angular momentum of a rigid body • Motion of a system • Moving frames of Reference (continued) • Motion of a rigid body (continued) • General motion of a rigid body • General equations of impulsive motion • Theory of Small Oscillations • Equations of Motion for small oscillations • Normal co-ordinates and normal modes of vibration • Systems with a few and may degrees of freedom • Hamiltonian Formulation, Transformations and Hamilton-Jacobi Theory • Hamiltonian Formulation • Phase-space • If the hamiltonian $H$ is independent of $t$ explicitly, prove that $H$ is constant and equal to the total energy of the system • Passage from the Hamiltonian to the Lagrangian • Ignoration of co-ordinates and Routh’s Procedure • Variational Methods (Hamilton’s principle etc.) • Derivation of Hamilton’s equation from the variational principle • Extension of Hamilton’s principle to 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analytic expression for a contact transformation • Sub-groups of Mathieu transformations and extended point transformation • Hamilton-Jacobi Theory • Hamilton-Jacobi equation • Hamilton-Jacobi equation for Hamilton’s characteristic function • The Hamiltonian being given by $H = \left(\frac{1}{2}m \right) \left(p^2 + (p^2 r^2) \right) - (\lambda/r)$ and to use Hamilton-Jacobi theory to solve Kepler’s Problem for a particle in an inverse square central force field • Harmonic oscillator problem as an example of the Hamilton-Jacobi method • Separation of variables • Action angle variables • Motion of Spinning Tops and Gyroscopes • Simple motion of a top (or Steady precession of a top) • General motion of a top • Steady motion • Stability Investigation • Gyroscopic compass • Mechanics of Continuous Media • Equation of motion for the vibrating string • Propagation of waves along a string • String as a limiting case of a system of particles • Lagrange’s equations for vibrating string.
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Theory of Relativity

- Classical Theory of Relativity: Speed of Light • Inertial Frame (Galilean Frame) • Galilean Transformations • Fictitious Force • Electrodynamics • Fizeau's Experiment • Michelson and Morley Experiment • Explanation of Negative Results (Null Results) • Lorentz Transformations • The New Concept of Space and Time • Postulates of Special Theory of Relativity • Lorentz Transformation Equations • Consequences of Lorentz Transformations • Time Dilation or Apparent Retardation of Rest • An Interesting Example of Time Dilation • Experimental Verification on Time Dilation • Simultaneity • Relativistic Formulae for Composition of Velocities • Relativistic Formulae for Composition of Accelerations • Relativity of Time: Proper Time • Lorentz Transformation Form a Group • Aberration (Relativistic Treatment) • Doppler's Effect • Confirmation of Doppler Effect • Relativistic Mechanics • Mass and Momentum • Newton's Laws of Motion • Measurement of Different Units • Experimental Verification of the Relation • Transformation Formula for Mass • Transformation Formula for Momentum and Energy • Particle with Rest Mass Zero • Binding Energy • Transformation Formula for Force • Relativistic Transformation Formula for Density • Minkowski Space (Four Dimensional Continuum) • Geometrical Interpretation of Lorentz Transformation • Space and Time Like Intervals • World Points and World Lines • Light Cone • Proper Time • Energy-Momentum Four Vector • Four Vector (World Vectors) • Relativistic Equations of Motion • Minkowski's Equation of Motion • Special Relativity in Classical Mechanics • Lorentz Transformation • Relativistic Lagrangian and Hamiltonian • Relativistic Hamiltonian
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- The Fundamental Theorem of Algebra.
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  - Integration by parts as applied to the functions of the type $e^x[f(x) + f'(x)]$.
  - Integrals of $e^x\sin bx$ and $e^x\cos bx$.

- Some Special Integrals

- Three special integrals:
  - $\int \frac{dx}{x^2 + a^2}$
  - $\int \frac{dx}{x^2 - a^2}$
  - $\int \frac{dx}{x^2 + a^2}$

- Evaluation of integrals of various types by using standard results.

- Three more special integrals:
  - $\int \frac{dx}{\sqrt{x^2 + a^2}}$
  - $\int \frac{dx}{\sqrt{x^2 - a^2}}$
  - $\int \frac{dx}{\sqrt{x^2 + a^2}}$

- Evaluation of integrals of various types by using standard results.

- Three more special integrals:
  - $\int \frac{dx}{\sqrt{x^2 + a^2}}$
  - $\int \frac{dx}{\sqrt{x^2 - a^2}}$
  - $\int \frac{dx}{\sqrt{x^2 + a^2}}$

- Evaluation of integrals of various types by using standard results.

- Integration of some special irrational algebraic fractions:
  - $\frac{1}{\sqrt{ax^2 + bx + c}}$
  - $\frac{1}{\sqrt{ax^2 + bx + c}}$
  - $\frac{1}{\sqrt{ax^2 + bx + c}}$

- Integrals of the type:
  - $\int \frac{dx}{a + bc \cos x}$
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  - $\int \frac{dx}{a \sin x + bc \cos x}$

- Integration of $P \cos x + Q \sin x + R \frac{\cos x}{a \cos x + b \sin x + c}$.

- Integration of $\sin^m x \cos^n x$.
...Contd: Advanced Mathematics for Pharmacists

The definite integral as the limit of a sum • Differential Equations of First Order and First Degree • Definitions • Differential equations of first order and first degree • Variables separable • Homogeneous equations • Equations reducible to homogeneous form • Linear differential equations • Equations reducible to the linear form • Exact differential equations • Integrating factors • Change of variables • Linear Differential Equations with Constant Coefficients • Definitions • Determination of complementary function (C.F.) • The Particular Integral (P.I.) • Particular integral in some special cases • To find P.I. when \( Q = e^{ax} \), where \( V \) is any function of \( x \) • To find P.I. when \( Q = e^{ax} + f(\alpha) = C \) • To find P.I. when \( Q = \cos \alpha \) or \( \sin \alpha \) and \( F(-a^2) = 0 \) • To find P.I. when \( Q = xV \), where \( V \) is any function of \( x \) • The operator \( \frac{1}{D - \alpha} \)

Differential Equations • Methods of solving simultaneous linear differential equations with constant coefficients • Number of arbitrary constants • Simultaneous equations of the form \( P_1 \frac{dx}{P} + Q_1 \frac{dy}{Q} + R_1 dz = 0, P_2 \frac{dx}{P} + Q_2 \frac{dy}{Q} + R_2 dz = 0 \), where \( P_1, P_2, Q_1, Q_2, R_1 \) and \( R_2 \) are functions of \( x, y, z \) • Geometrical interpretation of the differential equations \( \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \) • Linear Dependence and Independence of Solutions of Equations

Linear dependence and independence of solutions of an equation • Fundamental set of solutions • Wronskian • Theorem • Biometrics • Data Collection • Primary and Secondary data • Collection of primary data • Collection of secondary data • Limitations of secondary data • Census and Sampling • Population and Sample • Census and sample enquiry • Census versus sample enquiry • Fundamental principles of sampling theory • Method of sampling • Organisation of Data • Classification of data • Object of classification • Basis of classification • Classification according to attributes • Classification by variables • Frequency distribution • Sturge’s rule for number of classes and size of class interval • Cumulative frequency distribution • Diagrammatic Representation of Data • Importance and utility of diagrams • Limitations of diagrams • Rules for construction of diagrams • Types of diagrams • Graphical Representation of Data • The histogram • The frequency polygon • The frequency curve • Cumulative frequency curve or ogive • Graphs of time series or line graphs • Measures of Central Tendency • Objectives of average • Characteristics of a good average • Various measures of central tendency • Some special problems relating to arithmetic mean • Properties of arithmetic mean • Correcting incorrect values • Merits and demerits of arithmetic mean • Median • Calculation of median • Properties of median • Advantages of the median • Partition values • Graphical determination of median quartiles etc. • Quartiles • Mode • Calculation of mode • Determination of mode from mean and median • Measures of Dispersion • Objects and importance of dispersion • Characteristics for a satisfactory measure of dispersion • Absolute and relative measure of variation • Measures of dispersion combined standard deviation • Correcting incorrect values of mean and standard deviation • Coefficient of variation • Mathematical properties of standard deviation • Choice of suitable measure of dispersion • Measures of Skewness and Kurtosis • Skewness • Measure of skewness • Moments • Conversion of moments about an arbitrary origin into moments about mean • Utility of moments • Kurtosis • Measure of Kurtosis • Correlation and Regression • Types of correlation • Methods of determining correlation • Regression • Linear and non-linear regression • Regression lines • Another form of regression lines • Method of fitting regression lines • Probability • A priori or classical definition of probability • A posteriori or empirical probability • Algebra of events • Probability defined on events • Permutations and combinations • Probability of a simple event • Addition rule of probability • Addition rule, when events are not mutually exclusive • Independence and the multiplication rule • Conditional probability • Probability of at least one event • Odds in favour and odds against • Probability based on Bernoulli’s trials • Inverse probability • Baye’s theorem • Random Variable and Probability Distribution • Random variable • Probability distribution • Mean and variance of random variable • Binomial and Poisson distribution • Coefficients of the binomials • Characteristics of binomial distribution • Recurrence formula for the probabilities of binomial distribution • Poisson distribution • Characteristics of Poisson distribution • Recurrence formula for the probabilities of Poisson distribution • Normal distribution • Definition • Some properties of normal distribution • Standard form of the normal distribution • Area under the normal curve • Method of consulting table • Fitting of normal distribution • Statistical Inference • Population and sample • Parameter and statistic • Sampling distribution of the statistic • Standard error of the statistic • Utility of standard error • Statistical inference • Errors in hypothesis testing • Procedure of test of significance • Various tests of significance • Test of significance based on \( t \)-distribution • Test of significance based on \( F \)-distribution • \( \chi^2 \) - distribution • Tests based on \( \chi^2 \) distribution • Analysis of variance • Assumptions • The basic principle of Anova • Analysis of variance of one way classified data • Short-cut method • Coding method • Statistical Tables • Area Under The Standard Normal Curve • Ordinates of The Standard Normal Curve • Critical Values of \( t \)-distribution • Critical Values of \( \chi^2 \)-distribution • Percentage Points of The \( F \) Distribution (Upper 1% Points) • Percentage Points of The \( F \) Distribution (Upper 5% Points) • Value of \( e^{-m} \) • Logarithms • Antilogarithms • Appendix.
...Contd: Basic Mathematics for Chemists

In Terms of Any Three Non- coplanar Vectors $i, m, n$ • Vector Triple Product • Vector Triple Product is not Associative • Scalar Product of Four Vectors • Vector Product of Four Vectors • Reciprocal System of Vectors • Differentiation and Integration of Vectors • Vector Function • Scalar Fields and Vector Fields • Limit and Continuity of a Vector Function • Derivative of a Vector Function With Respect to a Scalar • Differentiation Formulæ • Derivative of a Function $f(A)$ • Derivative of a Constant Vector • Derivative of a Vector Function In Terms of Its Components • Some Important Results • Integration of Vector Functions • Some Standard Results • Gradient, Divergence and Curl • Partial Derivatives of Vectors • The Vector Differential Operator Del $(\nabla)$ • Gradient of a Scalar Field • Formulas Involving Gradient • Equipotential Surfaces or Level Surfaces • Directional Derivative of a Scalar Point Function • Tangent Plane And Normal To A Level Surface • Divergence of a Vector 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Two Unknowns (Cramer’s Rule) • System of Linear Non-homogeneous Equations In Three Unknowns (Cramer’s Rule) • Algebra of Matrices • Matrix • Special Types Of Matrices • Submatrices of Matrices • Equality of Two Matrices • Addition of Matrices • Properties of Matrix Addition • Properties of Multiplication of A Matrix By A scalar • Multiplication of Matrices • Properties of Matrix Multiplication • Positive Integral of A Square Matrix • Transpose of A Matrix • Symmetric and skew-symmetric Matrices • Conjugate of a Matrix • Transposed Conjugate of a Matrix • Hermitian And skew-Hermitian Matrices • Orthogonal and Unitary Matrices • Singular and Non-singular Matrices • Adjoint and Inverse of a Matrix • Adjoint of a Square Matrix • Inverse or Reciprocal of a Matrix • Linear Equations • Solving Systems of Linear Equations Using Inverse of a Matrix • Submatrix of a Matrix • Rank of a Matrix • Echelon Form of a Matrix • Elementary Operations or Elementary Transformations of a Matrix • Symbols to be Employed For The Elementary Transformations • Elementary Matrices • Vectors • Linear Dependence And Linear Independence of Vectors • Homogeneous Linear Equations • Some Important Conclusions About The Nature of Solutions Of The Equations $AX = 0$ • Working Rule For Finding The Solutions Of the Equation $AX = 0$ • Systems of Linear Non-homogeneous Equations • Condition For Consistency • Condition For A System of $n$ Equations in $n$ Unknowns To Have A Unique Solution • Working Rule For Finding The Solution Of the Equations $AX = B$ • Eigen values and Eigen vectors • Matric Polynomials • Characteristic Values and Characteristic Vectors of a Matrix • Certain Relations Between Characteristic Roots and Characteristic Vectors • Nature of The Characteristic Roots of Special Types of Matrices • The Process of Finding the Eigen values and Eigen vectors of a Matrix • Cayley-Hamilton Theorem • Diagonalisation of a Matrix • Introduction to Vector Space • Some Basic Concepts • Vector space • General Properties of Vector Spaces • Vector Subspaces • Algebra of Subspaces • Linear Combination of Vectors • Linear Sum of Two Subspaces • Linear Dependence and Linear Independence of Vectors • Basis of a Vector Space • Introduction to Tensors • Superscript and Subscript • Space of $n$-dimensions of Subspace • Curve in $n$-Dimensional Space • Einstein’s Summation Convention • Transformation of Coordinates • Kronecker Delta • Some Properties of Kronecker Delta • Scalars or Invariants • Contravariant and Covariant (Tensors of Order One) • Tensors of Order Two • Tensors of Higher order (or Higher Rank) • Some Properties of Tensors • Symmetric and skew-symmetric Anti-symmetric Tensor • Addition and Subtraction of Tensors • Functions, Limits and Continuity • Functions • Examples of Some Real Functions • Some Definition and Basic Concepts • Limit of a Function at a Point • Algebra of Limits • Some Important Expansions • Some Important Properties of Limits • Factorisation Method • Evaluation of a Limit When the Direct Substitution Gives The Indeterminate Form • Some Standard Limits • One Sided Limits i.e., Right Hand and Left Hand Limits • Limits at Infinity and Infinite Limits • Continuity • Discontinuity • Jump of a Function at a Point • Working Rule For Checking the Continuity of a Function $f(x)$ At A Point $a$ of its Domain • Cauchy’s Definition of Continuity • Differentiability • Relation Between Continuity and Differentiability • Differentiation • Increments • The Differential Coefficient • Some Standard Results • List of Standard Results to be Committed to Memory • Differential Coefficient of the sum of two Functions • Differential Coefficient of the Product of Two Functions • Differential Coefficient of the Quotient of Two Functions • Differential Coefficient of a Function • Hyperbolic Functions • Inverse Hyperbolic Functions and their Derivatives • Inverse Functions • Differential Coefficients of Inverse Trigonometric Functions • Trigonometric 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Number Theory


Contd...
...Contd: Number Theory
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Bio-Mathematics

Mathematical Aspects of Population Biology • Some Fundamental Concepts • Models • Mathematical Modelling • Formulation of a Mathematical Model • Solution of a Mathematical Model • Interpretation of the Solution • Types of Models • Limitation of Models • Areas of Modelling • Some Simple Mathematical Models • Mathematical Modelling in Biology or Bio-mathematics • Single Species Models • Stability and Classification of Equilibrium Points • Relationship between Eigen values and Critical Points • Single-species Models (Non-age structured) • Exponential Growth Model • Formulation of the Model • Solution and Interpretation • Limitation of the Model • Effects of Immigration and Emigration on Population • Logistic Growth Model • Solution and Interpretation • Limitation of Logistic Model • Extension of the Logistic Model • Single-species Models (Age Structured) • Continuous-time Continuous-age Scale Population Models • Discrete-time Discrete-age Scale Population Models • Density Dependent Model • Two-sex Models • Continuous-time Discrete-age Population Model • Mc Kendrick Approach to age Structure • Two Species Populations Models • Predator Prey Model • Secular equation for Determining Stability • Some Other Prey-predator Models • Two Dimensional Models and Competition Models • Two Dimensional Model Without Carrying Capacity • Two Dimensional Model with Carrying Capacity • Competition Models • General Continuous Model for Competition • Competition Model with Time Delays • Simple Competition Model • Mathematical Models in Epidemiology • Basic Concepts • SI Model • SIS Model with Constant Coefficient • SIS Model with coefficient is a function of time t • SIS Model with Constant Number of Carriers • SIS Model when the Carriers is a Function of time t • General Deterministic Model with Removal (SIR Model) • Epidemic Model with Vaccination • Biological Fluid Mechanics • Some Basic Concepts of Fluid Dynamics • Poiseuille’s flow • Model for Blood Flow • Properties of Blood • Bifurcation in an Artery • Pulsatile Flow of Blood • Tans-capillary Exchange • Sedimentation.

Cryptography and Network Security

Introduction to Security Attacks • Services and Mechanisms • Introduction to cryptography • Conventional Encryption: Conventional Encryption model • Classical encryption techniques • Substitution ciphers and transposition ciphers cryptographic • Steganography • Stream and block ciphers • Midem Block ciphers: Block Ciphers principles • Shannon’s Theory of Confusion and diffusion • Fiesta Structure • Data Encryption Standards (DES) • Strength of DES • Differential and Linear Cryptanalysis of DES • Block Cipher Modes of Operation • Triple DES • IDEA encryption and decryption • Strength of IDEA • Confidentiality using Conventional Encryption • Traffic confidentiality • Key distribution • Random number generation • Introduction to group • Ring and field • Prime and Relative Prime numbers • Modular arithmetic • Fermat’s and Euler’s Theorem • Primality Testing Euclid’s Algorithm • Chinese Remainder Theorem • Discrete Logarithms • Principles of public key cryptosystems • RSA algorithm • Security of RSA • Key management • Diffie-Hellman key Exchange algorithm • Idea of Elliptic Curve cryptography • Elgelam Encryption • Message Authentication and Hash Function: Authentication requirements • Authentication functions • Message Authentication codes • Hash functions • Birthday attack • Security of Hash function and MACSM, MD3 message digest algorithm • Secure Hash Algorithm (SHA) • Digital Signatures: Digital Signatures • Authentication Protocol • Digital Signature Standard (DSS) • Proof of digital signature algorithm • Authentication Applications: Kerberos and X.509 • Directory authentication service • Electronic Mail security-Pretty Good Privacy (PGP), S/MIME • IP Security: Architecture • Authentication Header • Encapsulating security payloads • Combining security associations • Key management • Web Security: Secure Socket Layer and Transport Layer Security • Secure Electronic Transaction (SET) • System Security: Intruders • Viruses and related threats • Firewall design principles • Trusted systems.
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—J.P. Chauhan

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Space Dynamics

—J.P. Chauhan

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Calculus of Variations


–A.R. Vasishtha & Vipin Vasishtha
–Mukesh Kumar Singh
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Krishna's

Fully Solved Series on

MATHEMATICS

for

(All Indian Universities and Competitive Examinations)

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~A.R. Vasishta & A.K. Vasishta

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- A.R. Vasishtha & S.K. Sharma

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- To Find the Perpendicular Distance of a Point P (x’, y’, z’) from a Line Through A (a, b, c) and whose Direction Cosines are
- The Plane
- The Equation of a Plane (Normal Form)
- To Prove that the General Equation of the first degree
- The Reduce the General Equation of the Plane to the Normal Form
- Intercepts Form
- Plane Through a Given Point and Perpendicular to a Given Line
- Equation of a plane through Three points
- Equations of the co-ordinate Planes
- Angle between Two Planes
- The Two Sides of a Plane
- To Find the Length of the Perpendicular from the Point to a Given Plane
- To Find the Distance between two Parallel Planes
- A Plane Through the Intersection of two Given Planes
- To Find the Condition that a Line
- The Angle between a Line and a Plane
- Equations of the Planes Bisecting the Angles between two Given Planes
- Combined Equation of a Pair of Planes
- Projection on a Plane
- Area of a Triangle
- The Straight Line
- General Equations of a Straight Line
- Symmetrical Form of the Equations of a Straight Line
- Line Through two Points
- To Transform the General form of the Equations of a Straight Line to Symmetrical Form
- The Plane and the Straight Line
- To Find the Equation of the Plane
- To Find the Equation of the Plane Through a Given Line and Parallel to Another Line
- Foot and Length of Perpendicular from a Point to a Line
- Co-planar Lines
- To Determine the Equations of a Straight Line Intersecting two Given Lines
- To Find Contd...
...Contd: Series : Analytical Geometry of Three Dimensions

the Perpendicular Distance of a Point from a Line and the Co-ordinates of the Foot of the Perpendicular ● Intersection of Three Planes ● Shortest Distance ● Skew Lines ● Length the Equations of the Line of Shortest Distance ● Volume of Tetrahedron ● To Find the Volume of a Tetrahedron, whose three Coterminal Edges in the Right-handed Orientation are \( a, b, c \) where \( a, b, c \) are Vectors ● To Find the Volume \( V \) of a Tetrahedron, in terms of the Lengths of Three Concurrent Edges and their Mutual Inclinations ● To Find the Volume \( V \) of a tetrahedron ● Skew Lines ● The Equations of Two Skew Lines ● Change of Axes ● Transformation of Co-ordinates ● Change of Origin (Translation of Axes) ● Change of Directions of Axes (Rotation of Axes) ● Relations between the Direction Cosines of three Mutually Perpendicular Lines ● The Sphere ● Equation of a Sphere ● Plane Section of a Sphere ● Intersection of two Spheres ● The System of Spheres Through a Given Circle ● The Intersection of a Straight Line and a Sphere ● The Equation of the Tangent Plane ● Plane of Contact ● Pole and Polar Plane ● Properties of the pole and the polar plane ● The Polar line ● The Angle of Intersection of Two Spheres ● Touching Spheres ● The Length of the Tangent ● The Radical Plane ● The Properties of the Radical Plane ● The Radical Line (or Radical Axis) ● Radical Centre ● Coaxial System of Spheres ● The Cylinder ● Right Circular Cylinder ● Tangent Plane to a Cylinder ● Enveloping Cylinder ● The Cone ● The Cone with the Vertex at the Origin ● The Line \( x = y = m = z = n \) ● To Find the General Equation of a Cone ● The Equation of the Cone with a Given Vertex and a Given Conic as Base ● To Find the Condition for the General Equation of the Second Degree to Represent a Cone and to Find the Co-ordinates of its Vertex ● The Tangent Line and the Tangent Plane to a Cone ● The Condition of Tangency ● The Reciprocal Cone ● The Angle between the Lines in which a Plane Cuts a Cone ● Three Mutually Perpendicular Generators ● Three Mutually Perpendicular Tangent Planes ● Right Circular cone ● The Enveloping Cone ● Central Conicoids ● The Ellipsoid ● The Hyperboloid of One Sheet ● The Hyperboloid of two Sheets ● The Tangent Plane ● The Condition of Tangency ● The Director Sphere ● The Polar Plane ● Properties of the Polar Planes and the Polar Lines ● Locus of Chords Bisected at a Given Point ● Normal to a Conicoid ● Number of Normals ● Cubic Curve Through the Feet of the Normals ● To Find the Equation of the Cone Through Six Concurrent Normals (The Six Normals Drawn from a Point to an Ellipsoid) ● Diametral Plane ● Conjugate Diameters and Conjugate Diagonal Planes ● The Relationship between the Co-ordinates of the Points \( P, Q, R \) where \( OP, OQ \) and \( OR \) are the Conjugate Semi-diameters of an Ellipsoid ● Properties of Conjugate Semi-diameters of an Ellipsoid.

Series: Modern Algebra

451-07

- A.R. Vasishtha & Kiran Vasishtha

- Mappings, Binary Compositions and Relations ● Functions or Mappings ● ‘Into’ and ‘Onto’ Mappings ● ‘One-one’ and ‘Many-one’ Mappings ● Inverse function ● Composite of Mappings ● Binary operation or Binary composition ● Types of binary operations ● Relation ● Equivalence relations ● Equivalence classes ● Partitions ● Fundamental Theorem on equivalence relations ● Partial order relations ● Groups ● Algebraic structure ● Group Definition ● Abelian group ● Finite and infinite groups ● Order of a finite group ● General properties of a group ● Definition of a group based upon left axioms ● Composition tables for finite sets ● Addition modulo \( m \) ● Multiplication modulo \( p \) ● Residue classes of the set of integers ● An alternative set of postulates for a group ● Permutations ● Groups of permutations ● Cyclic permutations ● Even and odd permutations ● Integral powers of an element of a group ● Order of an element of a group ● Homomorphism and Isomorphism of groups ● Complexes and subgroups of a group ● Intersection of subgroups ● Cosets ● Relation of congruence modulo ● Lagrange’s theorem ● Order of the product of two subgroups of finite order ● Cayley’s theorem ● Cyclic groups ● Rings ● Ring with unity ● Elementary properties of a ring ● Rings with or without zero divisors ● Integral domain ● Field ● Division ring or skew field ● Isomorphism of rings ● Subrings ● Subfields ● Characteristic of a ring ● Ordered integral domains ● Ideals ● Principal Ideal ● Principal Ideal ring ● Divisibility in an integral domain ● Polynomial rings ● Polynomials over an integral domain ● Vector Spaces ● General Properties of vector spaces ● Vectors subspaces ● Linear combination of vectors ● Linear span ● Linear sum of two subspaces ● Linear dependence and linear independence of vectors ● Basis of a vector space ● Finite dimensional vector spaces ● Dimension of a finitely generated vector space ● Dimension of a subspace ● Homomorphism of vector spaces or Linear transformations ● Isomorphism of vector spaces ● Direct sum of spaces ● Dimension of a direct sum ● Complementary subspaces ● Coordinates ● Rings (Continued) ● Divisibility of polynomials over a field ● Division algorithm for polynomials over a field ● Euclidean algorithm for polynomials over a field ● Unique factorization domain ● Quotient rings or Rings of residue classes ● Homomorphism of rings ● Maximal ideal ● Prime ideals ● Euclidean rings ● Normal Subgroups ● Conjugate elements ● Normalizer of an element of a group ● Class equation of a group ● Centre of a group ● Conjugate subgroups ● Invariant subgroups ● Quotient groups ● Homomorphisms of groups ● Kernel of a homomorphism ● Fundamental theorem on homomorphism of groups ● More results on group homomorphism.

Series: Vector Calculus

452-11

- A.R. Vasishtha & A.K. Vasishtha

- Multiple Products ● Scalar triple product ● Vector triple product ● Lagrange’s identity for four vectors ● Vector product of four vectors ● Reciprocal system of vectors ● Differentiation and Integration of Vectors ● Vector function ● Scalar fields and vector field ● Limit and continuity of a vector function ● Derivative of a vector function with respect to a scalar ● Differentiation formulae ● Curves in space ● Integration of vector functions...
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455-16

Series: Statics


456-19

Series: Dynamics


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Content: Series: Vector Calculus

- Gradient, Divergence and Curl
- Partial derivatives of vectors
- The vector differential operator \( \nabla \)
- Gradient of a scalar field
- Level surfaces
- Directional derivative of a scalar point function
- Tangent plane and normal to a level surface
- Divergence of a vector point function
- Important vector identities
- Green's, Gauss's and Stoke's Theorems
- Line integrals
- Surface integrals
- Volume integrals
- Green's Theorem in the plane
- The Gauss's divergence theorem
- Stoke's Theorem
- Line integrals independent of path.

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Introduction (Concurrent Forces, Lami's Theorem)
- Action and Reaction
- Resultant Force
- Parallelogram of Forces
- \( \lambda - \mu \) Theorem
- Components of a Force in Two Given Directions
- Resolved Parts of a Force Along two Mutually Perpendicular Directions
- Resultant of a Number of Coplanar Forces Acting at a Point
- Conditions of Equilibrium of a Number of Forces Acting at a Point
- Triangle Law of Forces
- Converse of the Triangle of Forces
- Lami's Theorem
- Polygon of Forces
- Equilibrium of a Rigid Body (Moments, Equilibrium of Coplanar Forces)
- Moment of a Force About a Point
- General Theorems of Moments
- Couple
- If Three Forces Acting in one Plane Upon a Rigid Body
- Theorem
- Necessary and Sufficient Conditions for Equilibrium of a Rigid Body
- Equation of the Resultant
- Equilibrium of a Rigid Body Under the Action of Three Forces Only
- Two Important Trigonometrical Theorems
- Virtual Work
- Displacement
- A Rigid Body
- Kinds of Displacement of a Rigid Body
- Rotation of a Rigid Body About a Point
- Position Vector of a Point After a General Displacement
- Work Done by a Force
- Work Done by a System of Concurrent Forces
- Work Done by a Couple During a Small Displacement
- Work Done by a System of Forces During a Small Displacement
- Virtual Displacement and Virtual Work
- The Principle of Virtual Work
- Forces which are Omitted in Forming the Equation of Virtual Work
- Application of the Principle of Virtual Work
- Strings in Two Dimensions (Common Catenary)
- The Catenary
- Intrinsic Equation of the Common Catenary
- Cartesian Equation of the Common Catenary
- Some Important Relations for the Common Catenary
- Sag of Tightly Stretched Wires
- Strings in Two Dimensions (Catenary of Uniform Strength and Strings Resting on a Smooth and Rough Plane Curve)
- Catenary of Uniform Strength
- Law of Variation of the Mass of String
- Equilibrium of a Light Inextensible String Resting on a Smooth Plane Curve
- Equilibrium of a Heavy Inextensible String on a Smooth Curve in a Vertical Plane
- Equilibrium of a Light Inextensible String Resting in Equilibrium on a Rough Plane Under the Action of no External Forces
- Equilibrium of a Heavy Inextensible String Resting in Limiting Equilibrium on a Rough Plane Curve Under the Action of no External Forces
- Stable and Unstable Equilibrium
- The Work Function
- Work Function Test for the Nature of Stability of Equilibrium
- Potential Energy Test for the Nature of Stability of Equilibrium
- z-Test for the Nature of Stability
- Stability of a Body Resting on a Fixed Rough Surface
- Centre of Gravity
- Determination of the C.G. by Integration
- Centre of Gravity of a Plane Area
- Centre of Gravity of a Solid of Revolution
- Centre of Gravity of Surface of Revolution
- Centre of Gravity when the Density Varies
- Use of Multiple Integrals to Find the Centre of Gravity of any Volume
- Equilibrium of Forces in Three Dimensions ([Central Axis] Excluding Wrenches)
- To Find the Resultant of any Given Number of Forces Acting on a Particle
- Necessary and Sufficient Conditions of Equilibrium of a Particle Under the Action of a System of Forces
- Reduction of a System of Forces to a Single Force and a Couple
- Necessary and Sufficient Conditions of Equilibrium of a Rigid Body Under the Action of a System of Forces at any Points of it
- Wrench
- Central Axis
- Characteristics of a Central Axis
- Wrench and Screw
- Invariants
- Conditions for a Single Resultant Force
- Equations of the Central Axis
- Computation of \( X, Y, Z, L, M, N \)
- Constrained Bodies
- Conditions of Equilibrium of a Rigid Body with one Point Fixed
- Conditions of Equilibrium of a Rigid Body with Two Fixed Points
- Forces in Three Dimensions (Screws and Wrenches; Null Lines and Null Planes)
- Null Lines, Null Plane and Null Point (Definitions)
- Null Lines
- To Find the Equation to Null Plane of a Given Point \( (a, b, c) \) Referred to Any Axes \( Ox, Oy, Oz \)
- To Find the Null Point of the Plane
- To Find the Condition that the Straight Line
- Conjugate Forces and lines (Def.)
- Screw, Pitch and Wrench (Definitions)
- To Find the Resultant Wrench of Two Given Wrenches
- Reciprocal Screws (Def.)
- Attraction
- The Law of Attraction (Newtonian Law of Gravitation)
- Attraction
- Attraction of a Rod
- Attraction of a Curvilinear Rod
- Attraction of a Thin Uniform Spherical Shell
- Attraction of a Solid Sphere
- Potential
- Relation between the Attraction and Potential
- If \( V \) be the Potential of an Attracting Mass \( M \), at any Point \( P (x, y, z) \), then
- Potential of a Finite Rod
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- Potential of a Circular Disc
- Potential of a Spherical Shell
- Potential of a Solid Sphere.
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Miscellaneous Laws of Forces ● Constrained Motion ● Motion in a Vertical Circle ● Some Important Results of the Motion of a Projectile to be Used in this Chapter ● Motion on the Outside of a Smooth Vetical Circle ● Cycloid ● Motion on a Cycloid ● Motion on the Outside of a Smooth Cycloid with its Axis Vertical and Vertex Upwards ● Simple Pendulum ● Oscillations of a Simple Pendulum ● Beat of a Pendulum ● The Second’s Pendulum ● Gain or Loss of Beats (time) by a Clock ● Central Orbits ● Differential Equation of a Central Orbit ● Rate of Description of the Sectorial Area ● Elliptic Orbit (Focus as the Centre of Force) ● Hyperbolic and Parabolic Orbits (Centre of Force Being the Focus) ● Velocity from Infinity ● Velocity in a Circle ● Given the Central Orbit, to Find the Law of Force ● Apse and Apsidal Distance ● Property of the Apse-Line ● Given the Law of Force, to Find the Orbit ● The Inverse Square Law (Planetary Motion) ● Newton’s Law of Gravitation ● Motion Under the Inverse Square Law ● Kepler’s Laws of Planetary Motion ● Deductions from Kepler’s Laws ● Some Important Geometrical Properties of an Ellipse ● Time of Description of an Arc of a Central Orbit ● To Find the Time of Description of a Given Arc of a Parabolic Orbit Starting from the Vertex ● To Find the Time of Description of a Given Arc of an Elliptic Orbit Starting from the Nearer end of the Major Axis ● To Find the Time of Description of a Given Arc of a Hyperbolic Orbit Starting from the Vertex ● Motion in a Resisting Medium (In a Straight Line Only) ● Terminal Velocity ● Motion of a Particle Falling Under Gravity ● Motion of a Particle Projected Vertically Upwards ● Projectiles ● The Motion of a Projectile and its Trajectory ● Latus Rectum, Vertex, Focuss and Directrix of the Trajectory ● Time of Flight, Horizontal Range and Maximum Height ● Velocity at any Point of the Trajectory ● Locus of the Focus and Vertex of the Trajectory ● Some Geometrical Properties of a Parabola ● Projections to Hit a Given Point ● Range and Time of Flight on an Inclined Plane ● Range and Time of Flight Down an Inclined Plane ● Envelope of the Trajectories with the Same Velocity of Projection ● Paricles Suffered to Describe Parabolic Paths ● Work, Energy and Impulse ● The Concept of Work ● Work Done by a Constant Force ● Work done by a Variable Force ● Units of Work ● Power ● Kinetic Energy ● The Work-energy Principle ● Conservative and Non-conservative Forces ● Potential Energy (P.E.) ● The Principle of Conservation of Energy ● The Principle of Conservation of Energy for the Motion in Plane ● The Principle of Conservation of Linear Momentum ● Impulse Definition When the Force is Constant ● D’Alembert’s Principle (And Equations of Motion of a Rigid Body) ● Motion of a Particle ● Motion of a Rigid Body ● D’Alembert’s Principle ● General Equations of Motion of a Body ● Lineum Momentar ● Motion of the Centre of Inertia ● Motion Relative to the Centre of Inertia ● Impulse of a Force ● An Important Rule ● General Equations of Motion Under Impulsive Forces ● Moments of Inertia ● Moments and Products of Inertia with Respect to Three Mutually Perpendicular Axes ● Some Simple Propositions ● Moment of Inertia of a Uniform Rod of Length 2a ● Moment of Inertia of a Rectangular Lamina ● Moment of Inertia of a Circular Wire ● Moment of Inertia of a Circular Disc ● Moment of Inertia of an Elliptic Disc ● Moment of Inertia of a Uniform Triangular Lamina about One Side ● Moment of Inertia of a Rectangular Parallellopiped about an Axis Through its Centre and Parallel to One of its Edges ● M.I. of a Spherical Shell (i.e., Hollow Sphere) about a Diameter ● M.I. of a Solid Sphere about a Diameter ● M.I. of an Ellipsoid ● Routh’s Rule ● Theorem of Parallel Axis ● Moment of Inertia of a Plane Lamina about a Line ● Principal Axes ● Motion about a Fixed Axis ● Moment of the Effective Forces about the Axis of Rotation ● Equation of Motion of the Body about the Axis of Rotation ● Moment of Momentum about the Axis of Rotation ● Kinetic Energy ● Compound Pendulum ● Time of a Complete Small Oscillation of a Compound Pendulum ● Simple Equivalent Pendulum ● Minimum Time of Oscillation of a Compound Pendulum ● The Centre of Suspension and the Centre of Oscillation of a Compound Pendulum are Convertible ● Reactions of the Axis of Rotation ● Centre of Percussion ● Centre of Percussion of a Rod ● Centre of Percussion (In General Case).
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Series: Numerical Analysis
A.R. Vasishtha, S.K. Sharma & Hemlata Vasishtha


Series: Hydrostatics
A.R. Vasishtha, A.K. Vasishtha